Information Sciences 296 (2015) 282–306

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Aperiodic triggering mechanisms for networked control systems

Magdi S. Mahmoud [a,*], Azhar M. Memon [b]

[a] Systems Engineering Department, King Fahd University of Petroleum and Minerals, Dhahran 31261, Saudi Arabia
[b] Center for Engineering Research, Research Institute (CER-RI), King Fahd University of Petroleum and Minerals, Dhahran 31261, Saudi Arabia

ARTICLE INFO

Article history:
Received 23 February 2014
Received in revised form 28 October 2014
Accepted 2 November 2014
Available online 8 November 2014

Keywords:
Event-triggered
Networked control system
Periodic triggering
Self-triggered

ABSTRACT

A survey is presented on the triggering mechanisms in Networked Control Systems (NCSs). These mechanisms can be classified as periodic and aperiodic, where aperiodic mechanisms can be further divided into event- and self-triggered schemes. We focus on aperiodic triggering schemes and cover most of the work done with an emphasis on the theoretical results. A glance at the existing work shows a need to organize the scattered theoretical results on the subject, which will provide a basis for interested researchers and also facilitate to visualize open problems.

1. Introduction

With an increasing trend of wired and wireless networked control loops, the demand to address the issues of computational power, communication load, and energy consumption has also increased. The standard implementations of feedback control over a network or embedded platform use periodic schemes, whereby sensing and/or actuation is done at equidistant samples of time. Although a mature systems theory exists for such methodology which eases the design and implementation, it causes an enormous waste of energy and communication capabilities, especially when there is no need for a corrective feedback signal. This translates into considering alternates to the periodic implementation, namely, event- and self-triggered (ET and ST) mechanisms.

Both ET and ST schemes comprise two elements, a controller and a triggering mechanism. This mechanism determines the next update time of the control law based on the previously sampled state information. Particularly, in former, the sensor (or controller) node determines on the basis of a comparison between the present state and a threshold, if the information to the controller (or actuator) should be sent. As compared with the periodic setting, this significantly reduces the amount of required communication, however, computational cost at the sensing node increases due to continuous monitoring of the plant state which is not well-suited for the battery powered sensor nodes. Furthermore, it requires a dedicated hardware to check the event condition. ST mechanism was introduced in [55] as a remedy to this problem. This scheme does not require continuous checking of the state, rather it predicts update time on the basis of previously sampled state and plant dynamics. Hence, ET mechanism is reactive and ST is proactive.

In literature, ET scheme is referred using various terminologies such as, event-based sampling, event-driven sampling, Lebesgue sampling, dead-band sampling, send-on-delta sampling, level-crossing sampling, and state-triggered sampling.

* Corresponding author.
E-mail addresses: msmahmoud@kfupm.edu.sa (M.S. Mahmoud), azharmehmood@kfupm.edu.sa (A.M. Memon).

http://dx.doi.org/10.1016/j.ins.2014.11.004
0020-0255/

The survey is organized as follows. Section 2 gives some mathematical preliminaries and notations used in the paper. The literature for ET methodology is presented in Section 3, while that for ST scheme is given in Section 4. A comparison is presented between these two in Section 5 followed by the conclusion in Section 6. For the reader's ease, possible future directions are given as remarks, the advantages and disadvantages are pointed out in the discussion at the end of each subsection, and the table given in appendix lists the works which deal with time-delay.

2. Preliminaries and notations

A continuous function $\alpha : [0, a) \to [0, \infty)$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$. It belongs to class $\mathcal{K}_\infty$, if $a = \infty$ and $\alpha(r) \to \infty$ as $r \to \infty$. Similarly, β is of class $\mathcal{L}$ if it is continuous and decreasing to zero. A function $\zeta : [0, \infty) \to [0, \infty)$ is said to be of class $\mathcal{G}$ if it is continuous and non-decreasing and $\zeta(0) = 0$. A continuous function $\gamma : [0, a) \times [0, \infty) \to [0, \infty)$ is said to belong to class $\mathcal{KL}$ if, for each fixed s, the mapping $\gamma(r, s)$ belongs to class $\mathcal{K}$ with respect to r and, for each fixed r, the mapping $\gamma(r, s)$ is decreasing with respect to s and $\gamma(r, s) \to 0$ as $s \to \infty$. Class $\mathcal{KK}$ functions are defined in the same fashion.

Local stability is defined when the initial state of the system lies close to the equilibrium point. When it can lie anywhere in the state space then the stability is defined as global. A system is said to be uniformly stable if its stability is independent of the initial time $t_0 \geqslant 0$. A system is said to be stable if for each $\epsilon > 0$, there exists a $\delta = \delta(\epsilon) > 0$ such that if $\|x(t_0)\| < \delta$ then $\|x(t)\| < \epsilon, \forall t \geqslant 0$. It is said to be asymptotically stable if it is stable and δ can be chosen such that if $\|x(t_0)\| < \delta$ then $\lim_{t\to\infty} x(t) = 0$. A system is said to be exponentially sable if there exists $\sigma, \lambda \in \mathbb{R}^+$ such that $\forall t \geqslant 0$ $\|x(t)\| \leqslant \sigma \|x(t_0)\| e^{-\lambda t}$. The state of a system is said to be ultimately bounded if there exist constants $\varepsilon, \varrho \in \mathbb{R}^+$ (ε defined as the bound) and for every $\eta \in (0, \varrho)$ there is a constant $T = T(\eta, \varepsilon) \in \mathbb{R}^+$ such that if $\|x(t_0)\| < \eta$ then $\|x(t)\| \leqslant \varepsilon, \forall t \geqslant t_0 + T$. A system is said to be Input-to-State Stable (ISS) if there exist a class $\mathcal{KL}$ function γ and a class $\mathcal{K}$ function α such that for any initial state $x(t_0)$ and any bounded input $u(t)$, state of the system satisfies $\forall t \geqslant t_0 \geqslant 0$ the following inequality,

$$\|x(t)\| \leqslant \gamma(\|x(t_0)\|, t - t_0) + \alpha\left(\sup_{t_0 \leqslant \tau \leqslant t} \|u(\tau)\|\right)$$

Consider a system with input–output relation given as $y = Hu$ for some mapping H. This mapping is said to be $\mathcal{L}_p$ stable if there exist a class $\mathcal{K}$ function α, defined on $[0, \infty)$ and a nonnegative constant μ such that,

$$\|(Hu)_\tau\|_{\mathcal{L}_p} \leqslant \alpha(\|u_\tau\|_{\mathcal{L}_p}) + \mu, \quad \forall \tau \in [0, \infty).$$

It is finite-gain $\mathcal{L}_p$ stable if there exist nonnegative constants ζ and μ such that,

$$\|(Hu)_\tau\|_{\mathcal{L}_p} \leqslant \zeta \|u_\tau\|_{\mathcal{L}_p} + \mu, \quad \forall \tau \in [0, \infty).$$

Here $\mathcal{L}_p$ denotes the p norm where $1 \leqslant p \leqslant \infty$.

Expectation operator and conditional expectation are denoted as E[.] and E[.|.], respectively.

3. Event-triggered network control

ET networked control (ETNC) or ET control (ETC) caught a great deal of attention by the end of last decade and plenty of work was done focusing the development of systems theory. A classification of this large number of control methods was presented by [54] and an appropriate generic model was introduced.

The general structure of ETNC system for sensor-controller communication is shown in Fig. 1. It consists of the plant, an event detector, an observer, and a control signal generator. When an event occurs, the event detector sends plant output to the observer. Here, an event refers to a situation whereby the output crosses a predefined threshold. The observer then computes state estimates and passes information to the control signal generator which generates the input signal for the process. The observer and control generator operate in open-loop between the events, therefore, the design of the generator is a central issue. In case all the states are available, full state vector is transmitted with the occurrence of an event. Also, controller and actuator can be connected over the network.

Fig. 2 shows the timing relationships for an ET scheme. The black rectangles on the timeline indicate when the control task is being executed. The time $\mathrm{T}_j = r_{j+1} - r_j$ is called the task period and it is the interval between any two consecutive invocations of the control task. D_j is the delay in jth job and it is the time between finishing and release time, i.e., $\mathrm{D}_j = f_j - r_j$.

We now present the survey for ETNC.

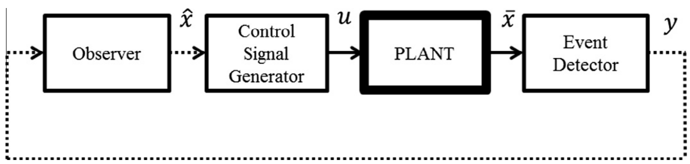

Fig. 1. Block diagram of event-triggered system. Solid lines denote continuous signal transmission and dotted lines show the event-based signals.

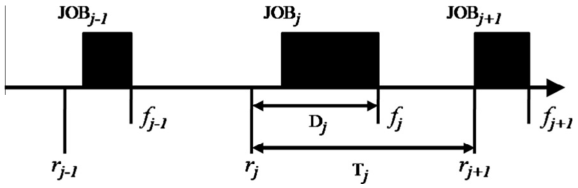

Fig. 2. Timing diagram for an event-triggered implementation.

3.1. Stability

Stability in terms of ultimate boundedness (UB) was studied in [18] for continuous-time (CT) linear systems with additive disturbances, on embedded systems. The authors provided first step in a proper analysis of these kind of loops and focused on the trade-off between performance and computational load to show the obtainable ultimate bounds and how they depend on the parameters of the control strategy. By using these results, the event-driven controller can be tuned to get satisfactory transient behavior and desirable ultimate bounds, while reducing the required average processor load. The theory is based on inferring properties of the ETC system from discrete-time (DT) linear systems (in case of nonuniform sampling) or piecewise linear (PWL) systems (in case of uniform sampling). The paper showed that even for a simple system, the complexity and challenge for analysis and synthesis is significant.

Before the stability results are given, we briefly define the concepts of uniform and non-uniform sampling in the context of ETC. Consider the LTI CT system given by,

$$\dot{x}(t) = A_c x(t) + B_c u(t) + E_c w(t), \tag{1}$$

where, $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the control input and $w(t) \in \mathcal{W}_c$ is the unknown disturbance. The set $\mathcal{W}_c \subset \mathbb{R}^p$ is convex and compact and contains the origin. Let $\mathcal{B}$ be an open bounded set containing origin, such that the control value is not updated until the state $x(t) \in \mathcal{B}$. In uniform sampling, it is checked every T_s time unit, whether the state lies in $\mathcal{B}$, i.e.,

$$\tau_{k+1} = \inf\{jT_s > \tau_k | j \in \mathbb{N}, x(jT_s) \notin \mathcal{B}\}, \tag{2}$$

where $\tau_k, k \in \mathbb{N}$ are the control update times. Nonuniform sampling, which is hard to implement in practice, does not require constant checking, rather it updates control whenever state reaches the boundary of $\mathcal{B}$, i.e.,

$$\begin{aligned} &\tau_1 = \inf\{t \geqslant \tau_o | x(t) \notin \mathcal{B}\}, \quad \text{and} \\ &\tau_{k+1} = \inf\{t \geqslant \tau_k + T_s | x(t) \notin \mathcal{B}\}, \quad k > 0, \end{aligned} \tag{3}$$

where $\tau_o = 0$ is the first control update time, irrespective of whether the initial state lies in $\mathcal{B}$.

A discrete-time state feedback controller with gain $F \in \mathbb{R}^{m \times n}$ is defined as

$$u_k = Fx_k, \tag{4}$$

where $x_k = x(\tau_k)$ and $u_k = u(\tau_k)$. Using zero-order hold (ZOH), $u(t) = u_k$ for all $t \in [\tau_k, \tau_{k+1})$. Hence, the system is given by

$$\begin{aligned} &\dot{x}(t) = A_c x(t) + B_c u(t) + E_c w(t), \\ &u(t) = Fx(\tau_k), \quad \forall \quad t \in [\tau_k, \tau_{k+1}). \end{aligned} \tag{5}$$

The control update times τ_k are equally spaced in time for periodic triggering mechanism and depend on state dependent condition for aperiodic schemes.

In the stability analysis of ETC scheme the discretized version of (1) and (4) for a fixed sampling time T_s is defined as,

$$x_{k+1}^d = (A + BF)x_k^d + w_k^d = A_{cl} x_k^d + w_k^d, \tag{6}$$

with

$$\begin{aligned} &A := e^{A_c T_s}, \\ &B := \int_0^{T_s} e^{A_c \theta} d\theta B_c, \\ &w_k^d := \int_{\tau_k}^{\tau_{k+1}} e^{A_c(\tau_{k+1} - \theta)} E_c w(\theta) d\theta. \end{aligned}$$

Moreover, for both uniform and nonuniform cases, system (5) behaves as (6) away from set $\mathcal{B}$.

The theorem given below, states UB for nonuniform sampling case.

Theorem 1. *For Nonuniform sampling: Consider system (5) and (3) with $\mathcal{W}_c$ a closed and convex set containing the origin. Let $\mathcal{W}_d$ be given as*

$$\mathcal{W}_d := \left\{ \int_0^{T_s} e^{A_c(T_s-\theta)} E_c w(\theta) d\theta | w \in \mathcal{L}_1^{loc}([0, T_s] \to \mathcal{W}_c) \right\}$$

(1) *If Ω is a robustly positively invariant (RPI) set for the linear discrete-time system (6) with disturbances in $\mathcal{W}_d$ and $cl\mathcal{B} \subseteq \Omega$, then Ω is RPI set for the event-driven system (5) and (3) on the control update times for disturbances $\mathcal{W}_c$.*
(2) *If system (6) with disturbances in $\mathcal{W}_d$ is UB to the RPI set Ω and $cl\mathcal{B} \subseteq \Omega$, then event-driven system (5) and (3) is UB to Ω for the disturbances in $\mathcal{W}_c$ on control update times.* □

For uniform sampling, the ETC is termed as periodic event-triggered control (PETC) [17] and the reader is referred to Section 3.6 for the stability theorem.

Asymptotic stabilization of linear systems with time-varying transmission delays was investigated by [37]. The sensor, controller and event-detector were considered to be collocated at a node. This allowed the event-generation to be control-error dependent rather than state-error. The authors provided with the criteria to design feedback gain and the ET mechanism which were driven to guarantee stability and performance.

Remark 1. The authors indicated discrete detection methodology (supervision of the event condition at discrete sampling instants), and joint design of event-detector parameter and the controller gain, as future works.

The problems of exponential stability, $\mathcal{L}_2$-gain analysis and $\mathcal{L}_2$-gain based controller design, along with network-induced delays and parameter uncertainties were studied in [19], using a unified model of NCSs with hybrid ET schemes. Sufficient conditions for exponential stability and $\mathcal{L}_2$-gain analysis were developed in the form of LMIs by using a discontinuous Lyapunov–Krasovskii functional approach. Moreover, two novel ET conditions were proposed: first on the sensor side, where an event-detector is placed between sensor and controller, which periodically (instead of continuously) checks the state to trigger an event, and second on the controller side, which decides when to send the control signal to the actuator.

Remark 2. The following topics deserve further investigation:

(1) To take into account the random communication delays and data packet dropouts and/or quantization, stochastic systems, and fault-tolerant control of NCSs with the proposed ET scheme,
(2) $\mathcal{H}_\infty$ controller analysis and synthesis for DT NCSs can be studied in the proposed framework, and
(3) Extension to $\mathcal{H}_\infty$ filtering.

A universal formula for event-based stabilization of general nonlinear systems affine in control, was proposed by [34]. It was proved that an event-based static feedback, smooth everywhere except at the origin, can be designed to ensure GAS of the origin. Also, for any initial condition within any given closed set, the minimal inter-sampling time is bounded from below, avoiding infinitely fast sampling phenomena, called Zeno behavior.

A framework to analyze stability and stabilization of ETC was proposed by [31], along with the tradeoff between communication and the desired performance. Lyapunov–Krasovskii functional approach was used for this purpose and sufficient criteria were obtained in terms of LMIs. The main feature of the scheme is that the released sampled data is not only determined by the current state and the error between the current and the latest transmitted state, but also by the current state of the network dynamics. Moreover, an information dispatching middleware was constructed which implemented novel ET scheme.

3.1.1. Discussion

The authors in [37] provided GAS guarantee for linear systems. Moreover, two limitations of this scheme are,

(1) Requirement of a delicate hardware for the event detector to monitor control signal and test the condition continuously, and
(2) The parameter of event detector was chosen with an assumption that the controller guarantees GAS without considering transmission delays.

The results of [34] for GAS of a class of nonlinear systems are based on the assumption of existence of a smooth control Lyapunov function (CLF). However, this assumption may limit the applicability of the results to a number of scenarios. For exponential stability, the methodology used in [19] can be extended to the case of parameter uncertainties.

The methodology of [31] considered network dynamics along with the control performance. In addition, the middleware allowed masking of the complex details of communication network, easing the control design.

3.2. Scheduling and event design

An ET scheme was presented in [65] which ensures exponential stability of the closed-loop system, and exploits the fact that the monotonic decrease of storage function is not necessary for stability of switched systems. An exponential function is properly chosen, with which, when the storage function intersects, the state is sampled. The inter-sampling periods were reported to be large as compared with the previous works. The ISS guarantee of CT system with respect to the measurement errors ensures that the inter-sampling periods and deadlines are bounded strictly away from zero.

A novel choice of the event function that only requires the computation of control and is independent of Lyapunov function, to ensure stability and non-zero inter-execution time for control-affine nonlinear systems was proposed and the strategy was used to stabilize an inverted pendulum by [10], which experimentally demonstrated a reduction of about 50% in the number of samples as compared with the periodic scheme.

3.2.1. Discussion

The event-design scheme in [65] was presented in the context of embedded-systems for nonlinear plants. The problems with ETC of nonlinear systems are the heavy computational demand of event function as compared with the control computation, and the use of Lyapunov function which is not necessarily available. For a class of nonlinear systems, [10] gave solution to both the problems. However, the control-based event functions did not consider delays.

3.3. Co-design

Simultaneous design of control and communication is referred to as co-design in the context of NCSs. There are two methodologies in the literature to achieve co-design of ETNC: Lyapunov approach [73,43], and cost function minimization which penalizes the inter-event times besides state and control variable [5–42]. Also, [20] discusses Lyapunov based co-design with state and input signal quantization and is mentioned in Section 3.7.

ET $\mathcal{H}_\infty$ control for the NCSs with uncertainties and transmission delays was considered in [73]. A delay system model was used which modeled delays and the event-driven system. Then, based on the model and Lyapunov functional method, and by using LMIs, the criteria for stability with an $\mathcal{H}_\infty$ norm bound and co-design, was given.

The scheme proposed in [43] maintains the desired $\mathcal{H}_\infty$ performance against disturbances, and takes into account the delays and packet loss. The main novelty was that the algorithm gives a *one-step* approach to co-design, as opposed to the previous methodologies which first design the controller with perfect signal transmission assumption, and then consider ET scheme.

A theoretical framework to analyze the trade-off between control performance and communication cost for transmission over a lossy network was presented in [5], where a controller-actuator network was considered. A multi-dimensional Markov chain model was used to represent packet retransmissions in case of packet loss. By combining this communication model with an analytical model of the closed-loop performance, a systematic way was provided to analyze the trade-off by appropriately selecting an event-threshold.

Co-design problem in a linear stochastic CT setting was considered in [38] by formulating the problem as the minimization of cost function which also penalizes the transmissions between sensor and controller,

$$J = \mathrm{E}\left[\int_0^T x_t' Q x_t + u_t' R u_t dt + x_T' Q_T x_T + \lambda k_T\right], \tag{7}$$

where Q, Q_T and R are standard matrices defined for optimal control problems, and the weighting factor $\lambda > 0$ penalizes $\mathrm{E}[k_T]$ which is the average number of transmissions in a finite interval of time $[0, T]$. The key innovation of this paper was to show that the underlying optimization problem is similar to two sub-problems, LQG regulator and optimal stopping time problems, hence enabling to use the standard techniques for optimal stochastic control to yield optimal ET policy. The optimization problem was reformulated in a way such that the separation principle is still valid. Numerical examples showed the effectiveness of the proposed scheme as compared to optimal time-triggered controllers.

Remark 3. Following topics deserve further investigations:

(1) The case of infinite horizon with discounted cost, where ergodicity issues are to be considered, and
(2) Extension to partial observations at the sensor-side, non-ideal communications and multi-terminal settings.

Now we present the results of [38] briefly. Consider the system given by,

$$dx_t = Ax_t dt + Bu_t + dw_t, \tag{8}$$

where $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times d}$. The initial state x_0 is given a priori at scheduler and controller. The vector valued Brownian motion process in $\mathbb{R}^n$ with zero mean and normalized variance is represented by w_t. Let k be the counting process with $k_0 = 0$, the value of which is incremented by one with every state transmission. The goal is to find the control policy u_t, and counting process k that minimize (7). The structure of optimal time-variant control law is given by,

$$u_t = \gamma^*(x_{\tau_k}, \tau_k, t) = -L_t \mathrm{E}[x_t|\mathcal{I}_t], \quad \tau_k \leqslant t < \tau_{k+1}, \tag{9}$$

where $\mathrm{E}[x_t|\mathcal{I}_t]$ denotes the expected value of the state given $\mathcal{I}_t = \{x_{\tau_k}, \tau_k\}$, the available information at the controller at time t. τ_k represent the stopping or transmission times and $\tau_0 = 0$. If τ_k is not defined, its upper bound is replaced by T, i.e., the time horizon. L_t is defined as,

$$
\begin{aligned}
L_t &= -R^{-1}B^T S_t, \\
-\frac{dS_t}{dt} &= A^T S_t + S_t A + Q - S_t B R^{-1} B^T S_t, \quad t \in [0, T],
\end{aligned} \tag{10}
$$

with initial condition $S_T = Q_T$. The estimation error is given as $\Delta_t = x_t - \mathrm{E}[x_t|\mathcal{I}_t]$ and at τ_k this error is zero, i.e., $\Delta_{\tau_k} = 0$. The complete design procedure is summarized in the following theorem.

Theorem 2. *The optimal ET controller minimizing (7) is given by,*

(1) *control policy (9) with L_T defined in (10),*
(2) *estimator,*

$$
\mathrm{E}[x_t|\mathcal{I}_t] = e^{(A-BL_t)(t-\tau_k)}x_{\tau_k}, \text{and} \tag{11}
$$

(3) *scheduling policy k^* which minimizes,*

$$
J^E(k) = \min_k \mathrm{E}\left[\int_0^T \Delta_t^T L_t^T R L_t \Delta_t + \lambda k_T\right], \tag{12}
$$

where Δ_t is a jump-diffusion process given as,

$$
\begin{aligned}
d\Delta_t &= A\Delta_t dt + dw_t, \\
\Delta_{\tau_k} &= 0,
\end{aligned} \tag{13}
$$

with initial condition $\Delta_0 = 0$. □

The co-design problem for multiple control systems closed over a common network was considered by [39]. Individual subsystems were modeled as DT stochastic linear systems with a quadratic control cost. The adaptation ability of event-based systems was exploited to develop a distributed algorithm, whereby each subsystem adjusts its ET mechanism to optimally meet the global communication network constraint, which was given by limiting the total average transmission rate of all subsystems. Numerical examples showed the effectiveness of the algorithm.

Remark 4. A few topics which deserve further investigations are:

(1) Convergence analysis of the overall adaptive system based on stochastic approximation, and
(2) Consideration of hard communication constraints rather than an average rate constraint with a limited number of transmission slots.

The certainty equivalence controller was reported to be optimal for an ET control system with resource constraints in [40]. The system model was an extension of the stochastic linear quadratic system framework. Three different types of resource constraints were considered: first one penalized every controller update with additional cost, second considered a limitation on the number of resource acquisitions, and third imposed a constraint on the average number of resource acquisitions. The obtained result is also valid in the presence of noisy measurements and for communication with delays and dropouts if instantaneous error-free acknowledgment channel exists.

Two suboptimal design strategies for ETC of linear DT stochastic systems in the presence of time-delays and packet-dropouts were presented in [41]. These strategies were based on certain design assumptions which made the separate design of controller and ET possible; due to these assumptions the strategies were regarded as suboptimal. Drift criteria, which is used to analyze asymptotic properties of Markov chains, was used for closed-loop stability analysis, which showed that sufficient conditions exist to guarantee bounded moment stability for both design strategies.

Remark 5. To analyze these algorithms for multiple control loops sharing a common communication network, where time-delays are varying and packet-dropouts have complex statistical models, can be considered for future research.

To decouple communication from control, [42] limited the usage of communication channel in terms of maximum allowable transmission rate, which depended upon the maximum number of transmission slots offered by the channel. The authors used the solution of stochastic optimal control problem in order to determine the optimal transmission rate for each system while optimizing the control cost.

Remark 6. Some avenues for further exploration are:

(1) To prove the *chaoticity in equilibrium* assumption for the underlying system,
(2) Realization of the centralized schedulers without gathering the state information of all heterogeneous multidimensional subsystems,

(3) Investigation on the online adjustment of the event-trigger according to the network traffic that also leads to a decentralization of the global resource allocation problem, and
(4) Studying more complicated models for the communication network.

3.3.1. Discussion

The proposed schemes in [73,43] use robust control with former being superior in terms of average release times compared with some other ET schemes in the literature, and later giving *one-step* approach for co-design.

In [40] it was indicated that the results cannot be extended to the case of ZOH control waveforms. In [41] numerical simulations indicated that the suboptimal procedures outperform time-triggered control systems, while marginally deviating from a lower bound on the system performance.

Decoupling of control from communication is a major issue in co-design methodology which was addressed by [42]. The framework thus provided determines the Pareto frontier for each subsystem offline, without the consideration of communication network parameters. Also, due to the consideration of a slotted protocol the approach can also be applied over a real-time slotted communication network. However, the authors did not consider rejection of the packets by the network controller, containing state information. As the methodology considered multiple control systems, [42] is also discussed in the context of ET-based decentralized systems in Section 3.8.1.

3.4. State feedback based ETC

A review of recent progress in utilization of ET scheme for state-feedback control was given in [25]. A state-feedback ETC scheme for which the performance of the closed-loop system approximates the behavior of a continuous state-feedback system with a focus on robustness against disturbance was proposed by [33]. The event based control policy was designed in such a way that the state of the closed-loop system remains in a bounded set around the state of the corresponding continuous time system. The extent of this bounded set, called the *approximation precision*, can be varied by changing the threshold of the event generator. Moreover, between two consecutive event times the unknown disturbance is estimated and a new event is only generated if the effect of the estimation error exceeds a given sensitivity bound. Hence, if the disturbance is small enough, no further event is generated.

This work was extended by [24] which improves the behavior of event-based control loop with respect to reference tracking and disturbance attenuation. It was shown that for a plant affected by a bounded, constant or time-varying disturbance, the scheme guarantees set-point tracking or holds the output in a bounded region around a prescribed reference signal, respectively. The experimental results demonstrated a considerable reduction in communication.

3.4.1. Discussion

The limiting assumptions of [33] are consideration of stable plant without uncertainties, delay free communication channel, and no restriction on the computational complexity. Although [24] considered reference tracking and disturbance attenuation, the overall approach is still not able to completely compensate for model uncertainties.

3.5. Output feedback based ETC and event based estimation

In many control systems all states of the plant are not available for measurement which motivates the study of ET systems with output feedback, and event-based estimation. In former, the plant output is either used to estimate the state on sensor side and then this estimate is transmitted to the controller, or the output is transmitted directly to the controller. In event-based estimation, sensor transmits the output to a *remote-observer* based on the occurrence of an event.

3.5.1. Output feedback control

An exhaustive analysis of the minimum inter-event time was not provided in the previous results for dynamical output-based ET control, which was addressed by [7] for decentralized setting (see Section 3.8.1), besides studying stability and $\mathcal{L}_\infty$ performance. Minimum inter-event time is guaranteed by choosing a triggering mechanism that depends on the difference between the output of plant or controller, and ITS previously sampled value plus a threshold. Furthermore, by modeling the ET system as impulsive system, closed-loop stability is guaranteed in terms of LMIs with larger minimum inter-event times than the existing results.

Tracking of an external reference for MIMO sampled plants with non accessible state, and their internal stability conditions were presented in [21]. Internal stability of the output feedback ET system was studied for the first time and the effectiveness of the method was shown, both in terms of tracking performance and reduced number of transmissions.

A dynamic output feedback based ETC scheme was introduced for stabilization of Input Feedforward-Output Feedback Passive (IF-OFP) NCSs, by [72]. The triggering condition derived based on the passivity theorem, not only allowed to characterize a large class of output feedback controllers but also showed the control system to be finite gain $\mathcal{L}_2$ stable in the presence of bounded external disturbances. The interactions between the triggering condition, the achievable $\mathcal{L}_2$ gain of the control system, and the inter-event time were studied in terms of the passivity indices of the plant and the controller. The same results were obtained with additional imperfections such as quantization of the transmitted signal and presence of the external disturbance.

When the triggering events in both sensor and controller only use local information to decide when to transmit the data, and the transmission in one link does not necessarily trigger the transmission in the other link, then the resulting ET scheme is referred to as *weakly coupled*. These type of triggering events in ET output feedback system with the control loop closed over a wireless network were presented in [28]. This represented an extension of the previous work, where the transmission from the controller subsystem was tightly coupled to the receipt of ET sensor data. An upper bound on the overall cost attained by the closed-loop system was given. Simulation results demonstrated that transmissions between sensors and controller subsystems are not tightly synchronized and were also consistent with derived upper bounds on the overall system cost.

The problem of dynamic output feedback controller (DOFC) design for CT LTI systems was addressed in [74]. In order to implement an output-based discrete ET condition, the methodology samples the output signal periodically instead monitoring it continuously. The closed-loop was modeled as a linear system with variable delays, and LMI based sufficient asymptotic-stability conditions were given which were used to co-design DOFC and the ET parameters. The authors assumed that the network does not have packet dropouts and disorders, however, their analysis accounted for bounded time-delays.

3.5.2. Event-based estimation

To decouple triggering and control, [48] introduced an event-based state estimator (EBSE) between the sensor and the controller. This EBSE provides the controller with periodic state estimates while it receives the plant data only at the event times, which translates into the state estimate update both at the event times and at periodic update instants. Until the EBSE receives the plant's output, the estimation is done based on the knowledge that the output remains in a bounded set that characterizes the event. These characteristics of the EBSE allow its estimation error covariance matrix to be bounded. These bounds are then translated into a polytope to be fed into a robust MPC algorithm which optimizes the closed-loop trajectory dependent ISS gain. Hence, the more accurate the received information by estimator, the better the trade-off between event generation and closed-loop performance.

Remark 7. As highlighted by the authors, the formal proof of closed-loop properties of the EBSE-MPC-plant interconnection or its variations is not yet available.

Distributed ET estimation over WSN was reported in [68], with the objectives to minimize sensor's energy consumption and network congestion while guaranteeing estimator's performance. Several sensor nodes transmit the parts of the state to a central estimator based on the ET policy which depends only on the local sensory information. The proposed scheme does not require sensors to *broadcast* their measurements, which drops the need for sensor nodes to continually listen to the wireless channel, thus saving a considerable amount of energy. Furthermore, congestion is avoided by allowing the sensors to receive information from the central estimator, which informs a sensor not to transmit its state in case it already has sufficient state information. The scheme was implemented to estimate water level in a six-tank system using two wireless sensor nodes to demonstrate the decrease in sensor transmissions and network congestion.

Remark 8. For future extensions of this work, it has been proposed to:

(1) Evaluate the proposed mechanism in a real WSN to see the effect of delaying or avoiding transmissions in high-traffic conditions, and
(2) Study the inclusions of a priori scheduling for estimator-sensor communication to guarantee state broadcast such that the probability of transmitting immediately after listening is reduced.

Communication rate analysis for the estimation quality was presented in [69,47]. Specifically in former, a sensor data scheduler with an accurate minimum mean squared error (MMSE) state estimator was proposed for linear systems. The estimator was based on an approximation technique for nonlinear filtering, which gave a relationship between sensor-estimator communication rate and estimation quality. It was shown that a slightly increased tolerance of the estimation error gave a significant reduction in the communication rate.

In [47], a remote estimator sends the predicted state to the scheduler (over a wireless channel), which implements a level-based ET condition depending upon the difference between the predicted state and the actual output. For the expected value of communication rate, an exact measure was provided for scalar valued sensor measurements, and upper and lower-bounds for vector valued measurements.

Remark 9. As an extension, one can consider other ET schemes such as time-dependent conditions that are designed to guarantee the estimation performance, and send-on-delta triggering conditions which do not require feedback communications.

3.5.3. Discussion

[21,28] consider a delay free communication channel.

For decoupling of communication and control, which is one of the major issues in NCSs design besides energy and bandwidth economy, [48] provides one way by introducing an ET-based estimator. [68] discusses the issue of the energy economy over WSN.

3.6. Periodic Event-Triggered Control (PETC)

In conventional ETC (termed as continuous ETC (CETC) in [17]), the ET condition is *continuously* monitored, which requires a dedicated analog circuitry, and infinitely fast sampling may occur. To tackle these problems, [17] proposed a scheme whereby the transmissions and controller computations are event-based while the ET condition is checked *periodically*, hence retaining the low resource utilization property of conventional ETC while guaranteeing a minimum inter-event time of at least one sampling period of the ET condition.

To analyze the stability and $\mathcal{L}_2$-gain properties of the resulting PETC systems, two different approaches were presented by the authors of [17] in previous works, based on (i) discrete-time PWL systems, and (ii) impulsive systems. Besides, techniques to compute (tight) lower bounds on the minimum inter-event times were also given. Numerical example showed that PETC is able to reduce communication and computation resource utilization significantly.

The authors extended their work to output-based dynamic controllers, and decentralized ET conditions in [17]. Moreover, a third approach (other than PWL and impulsive systems) namely, DT perturbed linear (PL) systems was presented, and compared with the aforementioned approaches. The PWL system approach provides least conservative LMI-based results in case of stability analysis only, the PL system approach has the lowest computational complexity and provides useful insights for emulation-based controller synthesis, while the impulsive system approach provides a direct $\mathcal{L}_2$-gain analysis of the system.

In what follows, we give the stability results for PETC systems using PWL systems approach. Let $\xi(t) = [x^T \quad \hat{x}^T]^T \in \mathbb{R}^{n_\xi}$, where $\hat{x}$ is the latest transmitted state. When PETC is implemented on system (1), then control law takes the form,

$$u(t) = K\hat{x}(t), \quad \text{for} \quad t \in \mathbb{R}_+, \tag{14}$$

where, $K \in \mathbb{R}^{m\times n}$. Let τ be the factor that keeps track of the time elapsed since last sampling time and $h > 0$ be a properly chosen sampling interval, and define,

$$\overline{A} := \begin{bmatrix} A_c & B_cK \\ 0 & 0 \end{bmatrix}, \quad \overline{B} := \begin{bmatrix} E_c \\ 0 \end{bmatrix}, \quad J_1 := \begin{bmatrix} I & 0 \\ I & 0 \end{bmatrix}, \quad J_2 := \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix},$$

to give an impulsive system,

$$\begin{aligned} \begin{bmatrix} \dot{\xi} \\ \dot{\tau} \end{bmatrix} &= \begin{bmatrix} \overline{A}\xi + \overline{B}w \\ 1 \end{bmatrix}, \quad \text{when} \quad \tau \in [0, h], \\ \begin{bmatrix} \xi^+ \\ \tau^+ \end{bmatrix} &= \begin{bmatrix} J_1\xi \\ 0 \end{bmatrix}, \quad \text{when} \quad \xi^T Q\xi > 0, \tau = h, \\ &= \begin{bmatrix} J_2\xi \\ 0 \end{bmatrix}, \quad \text{when} \quad \xi^T Q\xi \leqslant 0, \tau = h, \\ z &= \overline{C}\xi + \overline{D}w, \end{aligned} \tag{15}$$

where $Q \in \mathbb{R}^{n_\xi \times n_\xi}$ is some symmetric matrix and z is the performance output with appropriately chosen $\overline{C}$ and $\overline{D}$.

Let $t_k = kh, k \in \mathbb{N}$ be the periodic sampling times then whether or not the new state measurements are transmitted to the controller is based on the ET condition $\mathcal{C}$ given by,

$$\mathcal{C}(\xi(t_k)) = \xi^T(t_k)Q\xi(t_k) > 0, \tag{16}$$

rendering $\hat{x}(t)$ as,

$$\begin{aligned} \hat{x}(t) &= \; x(t_k), \quad \text{when} \quad \mathcal{C}(x(t_k), \hat{x}(t_k)) > 0, \\ &= \; \hat{x}(t_k), \quad \text{when} \quad \mathcal{C}(x(t_k), \hat{x}(t_k)) \leqslant 0. \end{aligned} \tag{17}$$

To obtain a complete PETC system (1), (14), (16) and (17) are combined.

We first define PWL system formulation for PETC system (15) and then present the stability result, as given in [17]. By discretizing (15), a bimodal PWL system is obtained as,

$$\begin{aligned} \xi_{k+1} &= \; A_1\xi_k, \quad \text{when} \quad \xi_k^T Q\xi_k > 0, \\ &= \; A_2\xi_k, \quad \text{when} \quad \xi_k^T Q\xi_k \leqslant 0, \end{aligned} \tag{18}$$

where

$$A_1 := e^{\bar{A}h}J_1 = \begin{bmatrix} A+BK & 0 \\ I & 0 \end{bmatrix}, \quad A_2 := e^{\bar{A}h}J_2 = \begin{bmatrix} A & 0 \\ I & 0 \end{bmatrix},$$

with

$$A := e^{A_c h}, \quad B := \int_0^h e^{A_c s} B_c ds. \tag{19}$$

Using PWL system (18) and a piecewise quadratic (PWQ) Lyapunov function of the form,

$$\begin{aligned} V(\xi) = &\ \xi^T P_1 \xi, \quad \text{when} \quad \xi_k^T Q \xi_k > 0, \\ = &\ \xi^T P_2 \xi, \quad \text{when} \quad \xi_k^T Q \xi_k \leqslant 0, \end{aligned} \tag{20}$$

and taking $w = 0$, following stability theorem is defined:

Theorem 3. *PETC system (15) is globally exponentially stable (GES) with a decay rate ρ, if there exist matrices P_1, P_2, and scalars $\alpha_{ij} \geqslant 0, \beta_{ij} \geqslant 0$ and $\kappa_i \geqslant 0, i,j \in \{1,2\}$, satisfying*

$$e^{-2\rho h} P_i - A_i^T P_j A_i + (-1)^i \alpha_{ij} Q + (-1)^j \beta_{ij} A_i^T Q A_i \geqslant 0, \tag{21}$$

for all $i,j \in \{1,2\}$, and

$$P_i + (-1)^i \kappa_i Q > 0, \tag{22}$$

for all $i \in \{1,2\}$. □

3.7. Quantized systems

Besides using aperiodic triggering techniques to reduce the amount of traffic over the network, another way is to use *quantized* state (for sensor-controller (SC) network) and/or input (for controller-actuator (CA) network) variables. However, quantization also has adverse effects besides decreasing the number of bits used to represent the state/input information. [71,70,20,30] deal with these effects.

The problems of state feedback as well as output feedback control of NCSs in the presence of network imperfections such as delays, dropouts, and quantization were addressed in [71]. The methodology compensates for the effect of random packet dropouts. Sufficient conditions were also given for the asymptotic stability of the overall system. A similar work was presented by the authors in [70] which considered periodic SC, and an event-based CA network. The focus was on delays in the communication channel in both the feedback (SC), and the forward (CA) loops. The methodology introduced a predictive-control methodology to deal with packet delays, dropouts and disorders, whereby the control sequence is predicted with a prediction horizon which depends on the bounds of the delays, and sent to the actuation side.

The problem of ET co-design of CT linear networked systems with delays, and both state and control input quantization was studied in [20]. A new delay model was used to represent the overall system to take into account ET mechanism, delays, and quantization in a unified framework. Sufficient conditions for asymptotic stability were given in terms of LMIs using Lyapunov–Krasovskii functional approach. Moreover, in order to achieve a better trade-off between control performance and network conditions, an explicit expression for the feedback gain was given with integration of signal quantization levels, delays and trigger parameters.

Remark 10. The authors considered infinite quantization levels, hence to merge finite number of quantization levels and ET scheme in a unified framework can be considered as possible extension.

According to [30], a more realistic measure of channel usage for ET scheme, as compared with the inter-transmission interval, is the *stabilizing bit-rate*, which is defined as the number of bits per sampled state divided by the acceptable delay in message delivery. The bit-rates required to asymptotically stabilize nonlinear ET systems were examined, with quantization effects, and maximum acceptable delays. These imperfections might render the bit rates required by the ET scheme to be greater than those required by periodic scheme. Scaling relationships between maximum delay, inter-sampling interval, and quantization error were used to bound the stabilizing bit-rates. Conditions were presented under which the stabilizing bit-rate asymptotically goes to a constant value, and in some cases to zero as the system approaches equilibrium.

Remark 11. Considering the available literature, the following topics are possible extensions of this work:

(1) Consideration of system noise, and
(2) To study the scheduling problem when there are several controllers sharing the same communication network.

3.7.1. Discussion

The main limitation of [71] was the authors did not consider aperiodic communication. Moreover, in [70],

- an explicit analysis or expression, which indicates event-driven CA channel, was not given.
- the bounds on forward and feedback delays, which are directly related with the prediction horizon of control sequence, were not determined.
- the control performance depends on control sequence prediction accuracy.

The shortcoming pointed out in [20] was the use of infinite quantization levels.

3.8. ETC of decentralized and distributed systems

Ever increasing consumer demands have pushed today's industry to implement *cyber-physical* systems, whereby large scale dynamical systems consisting of a number of coupled subsystems use a communication network for control and monitoring purpose. Examples are process industry and electric power supply companies, to name a few. Control of these systems in a centralized fashion imposes stringent demands on the communication medium and modeling of subsystem coupling, necessitating the use of decentralized [36,7,67,42] or distributed [6,14,15,32,59,60,62,66] networked control strategies using aperiodic communication schemes. Former methodology is applicable when the subsystems have weak or no coupling, and later is implemented otherwise. In what follows, we present a survey on both implementing ET scheme.

3.8.1. Decentralized systems

Decentralized ET implementation of centralized nonlinear controllers over Wireless Sensor Actuator Networks (WSAN) was presented in [36], without assuming weak-coupling between the systems. The main motivation for decentralized implementation comes from the fact that the sensor nodes are physically distributed in WSANs which implies that all measured quantities are not accessible, translating into the usage of observer based techniques; this is impractical due to low computational capabilities of sensor nodes. Moreover, the consensus based techniques demand a large amount of communication. The proposed computationally efficient methodology relied only on the local information and offered large controller inter-computation times while guaranteeing performance. As mentioned by the authors, the techniques can be implemented over WirelessHART standard.

Dynamic output-based ET controllers were proposed by [7] in a decentralized setting. Due to the physical distribution of sensors and actuators (grouped into nodes), and controllers, the continuous transmission of state and control variables for ET conditions is not possible. This problem is solved by a decentralized ET mechanism whereby the events are based only on the local information. In order to guarantee a positive minimum inter-event time, the event occurs when the difference between the current value of a node and its previously transmitted value becomes larger than the current value plus an additional threshold. Moreover, the ET system was modeled as an impulsive system. Closed-loop stability and $\mathcal{L}_\infty$ performance were guaranteed along with larger inter-event times than the previously existing results.

The idea of asynchronous decentralized ET transmission was adopted by [67]. The proposed methodology allows asynchronous transmission of the state information from geographically distributed sensor nodes, thus relaxing the consistency requirement at the controller. Correspondingly, the controller output is also computed and actuated in an asynchronous manner. Asymptotic stability of the overall NCS is guaranteed if the weighted sum of all minimal transmission periods and all types of delays is bounded, which translates into a tradeoff between the system performance, and overall communication and computational resources. Additionally, strictly positive minimal transmission periods were provided.

Multiple heterogeneous control systems sharing a common communication medium to close the feedback loop were considered in [42]. Using an approximate formulation for the communication medium enables to divide the overall optimization problem into two levels: (1) a local average-cost problem for every subsystem solved using dynamic programming, and (2) a global resource allocation problem for optimal transmission rates, which is a convex optimization problem. It was shown that the overall system is stochastically stable and that the system is asymptotically optimal as the number of subsystems is increased. The numerical examples showed an increased level of flexibility, robustness, and a significant improvement in the control performance.

Remark 12. Considering available literature, following are the open problems for further research:

(1) To investigate online adjustment of the event-trigger according to the network traffic that also leads to a decentralization of the global resource allocation problem, and
(2) To study complicated models for the communication network.

The stochastic stability results due to [42] are presented as follows. Consider a networked control system with N control loops closed over a common communication medium. The process $\mathcal{P}^i$, where $i \in \{1, \ldots, N\}$ is described by,

$$x^i_{k+1} = A^i x^i_k + B^i u^i_k + w^i_k, \tag{23}$$

with state $x_k^i \in \mathbb{R}^{n_i}$, input $u_k^i \in \mathbb{R}^{d_i}$ and i.i.d noise process $w_k^i \in \mathbb{R}^{n_i}$. The initial state x_0^i is a random variable with symmetric distribution around its mean and finite second order moment. The statistics of the random variables and the system parameters within a subsystem are known to the controller and sensor station. The number of transmission slots are limited to N_{slot} because of the limited bandwidth, translating into the design of priority based ET mechanism on sensor nodes, whereby the node decides on the importance of sending the data.

The optimal control law is given as,

$$\begin{aligned} u_k^i &= \gamma_k^{i,*}(Z_k^i) = -L^i \mathrm{E}[x_k^i | Z_k^i], \\ L^i &= (B^{i,T} P^i B^i + Q_u^i)^{-1} B^{i,T} P^i A^i, \\ P^i &= A^{i,T}(P^i - P^i B^i (B^{i,T} P^i B^i + Q_u^i) B^{i,T} P^i) A^i + Q_x^i, \end{aligned} \tag{24}$$

with Z_k^i representing the observations at the controller side.

The conditions that guarantee stochastic stability of the aggregate system are given in the following theorem:

Theorem 4. *Let the control law be given by (24) and let the scheduling policy be $\pi_k^i(\delta_k^i = 1 | e_k^i) = 1$ for $\|e_k^i\|_2 > M^i$ for some arbitrary M^i. If*

$$\frac{N_{slot}}{N} > 1 - \frac{1}{\|A^i\|_2^2}, \tag{25}$$

is satisfied for all subsystems, then the Markov chain representing the aggregate system is stochastically stable. □

3.8.2. Distributed systems

The previous results on ET broadcasting of state information in distributed control systems over wireless networks were extended by [59] for a network consisting nonlinear CT systems. The methodology also considered delays and packet dropouts. The main contribution was to completely decentralize the ET scheme implying that,

- subsystem broadcasts its information using only local data,
- only individual subsystem's and its immediate neighbors' information is required to determine the triggering threshold,
- the deadline for subsystem's broadcast can be anticipated based only on local information, and
- subsystem can locally identify the maximum allowable number of data dropouts.

The overall system under the proposed scheme is guaranteed to be GUUB if the transmission delays are lower bounded by a strictly positive constant, for a limited number of successive data dropouts assumption for each subsystem.

The work in [59] served as preliminary for [66], which gave an analysis that is applicable for both nonlinear and linear subsystems. In former, the event design was transformed into a local ISS design problem, while for later, the design reduced to a local LMI feasibility problem. NCS was shown to be finite-gain $\mathcal{L}_p$ stable for zero-delay, and bounded successive data dropouts assumptions. For non-zero transmission delay (less than the corresponding deadline), and if the external disturbance vanishes, the NCS is asymptotically stable. Simulations revealed that the average broadcast period and the computational time required to select thresholds have a good scalability with respect to the system size.

As compared with [59,66], results for ET distributed setting applicable to a very large class of systems were presented in [6]. The ET scheme, which depends only on the local information, introduces some disturbances in the system. This translates into a modification of general small-gain theorem because the ISS small-gain results are not applicable. It was assumed that the interconnection terms satisfy a generalized small-gain condition, and the graph modeling of the system is strongly connected. Furthermore, the infinite sampling phenomenon was mitigated using either input-to-state practical stability (ISpS), or Lyapunov function approach. This novel methodology was termed as *parsimonious triggering*, as it reduces the number of necessary events.

Remark 13. To avoid collision, whereby events occur simultaneously at multiple subsystems, and to derive explicit bounds of inter-event times can be considered as future extensions of this work.

The authors in [15] used a model-based approach in which each subsystem contains a model of its neighboring nodes, thus reducing the amount of broadcasted events. Additionally, an analysis was presented for the effect of interaction over the region of convergence around the equilibrium, and the state independent strictly positive lower bound of the broadcasting period. Furthermore, the effects of network induced delays and packet losses were considered. Two different communication protocols were proposed which guarantee stable behavior in the presence of network imperfections. The analysis thus presented provides with bounds on the delays, and the number of successive dropouts which ensure stability and performance, while giving lower bounds on the inter-event times.

For the similar setup, imperfect decoupling was considered by [14]. A novel ET mechanism was proposed which considers time-dependent trigger functions to guarantee asymptotic stability, and existence of a strictly positive lower bound for the inter-event time. Moreover, the model uncertainties were taken into account in the inter-event time analysis.

Remark 14. The authors highlighted to consider the following as future extensions of their work:

(1) Application of the proposed methodology to DT systems, and
(2) Consideration of exogenous disturbances.

A widely used distributed algorithm that solves network utility maximization (NUM) called *dual decomposition algorithm*, was compared with ET distributed algorithms by [60,62]. State-dependent ET thresholds under which the distributed NUM algorithm, based on barrier methods, converges to the optimal solution of the NUM problem, were established by [60]. Simulation results suggested that the proposed algorithm reduces the number of message exchanges by up to two orders of magnitude when compared with dual decomposition algorithms, and is independent of the maximum path length or maximum neighborhood size (measures of network size). A general class of optimization problems was given in [62], where an ET distributed algorithm was used for sensor networks. As an example, the authors used data gathering problem and showed that the proposed algorithm reduced the number of message exchanges by an order of two as compared with dual decomposition algorithm. Additionally, the methodology was independent of the network size.

The problem of designing an appropriate distributed ET rule to achieve asymptotic synchronization of a dynamical network with linear subsystems while avoiding Zeno-behavior was addressed by [32]. The complexity of the problem lies in the limited information constraint, whereby a subsystem has access only to its neighbor's information available at discrete instants. To overcome this problem, estimators were introduced in each subsystem which provided an estimate of the neighboring nodes' information using the limited information. The network was shown to achieve asymptotic synchronization without Zeno-behavior for all time.

Remark 15. The methodology did not consider network imperfections such as quantization, delay, and dropouts, providing an interesting avenue to explore.

3.8.3. Discussion

The methodology of [36] does not use asynchronous decentralized ET control. Moreover, co-located sensor nodes translate into the need of their coordination and information sharing.

The assumptions of weak-coupling and restrictive-dynamics are imposed in [7], which considers linear systems. The work presented in [42] is also discussed in Section 3.3.

In the context of distributed systems, the main shortcoming of [66] is the occurrence of infinitely fast data transmission when the system reaches the origin because a lower bound for inter-broadcast times is not guaranteed.

The methodology presented in [60,62] does not consider network artifacts.

3.9. Adaptive control

$\mathcal{L}_1$ adaptive control technique guarantees closed-loop system's stability with a very high degree of robustness. Real-time implementation of output-feedback $\mathcal{L}_1$ adaptive controller was reported in [58]. Stability conditions in terms of event threshold and allowable transmission delays were also provided.

Adaptive state feedback ETC of SISO affine nonlinear DT systems was presented in [45]. The knowledge of nonlinear system dynamics was partially relaxed by using a NN-based adaptive estimator which estimated the parameters as well as the states. The weights of NN were adjusted at aperiodic instants using ET scheme. Similar work was reported in [46] for uncertain DT linear systems identified as autoregressive Markov (ARMarkov) representation, for which an update law was derived to estimate the parameters at ET instants. After the convergence of parameter estimation error and output to zero, no more triggering is required.

3.10. Model based ETC

Model-based NCS (MB-NCS) and ETC were combined by [13], model-based event-triggered (MB-ET) control, which gave increased inter-update times as compared with the individual control strategies. In the proposed framework, nominal model of the plant is stored in the controller node which generates state estimates of the system during update intervals, hence giving better results as compared with the traditional ZOH version of state, and providing stability thresholds that are robust to model uncertainties. This work also considers quantization and time-varying network delays. The error events, designed based on the quantized measured variables, ensure asymptotic stability as opposed to the similar results which considered non-quantized measurements.

Remark 16. Future direction pointed out by the authors is to consider output feedback using the proposed framework.

PETC (see Section 3.6) was combined with MB strategy for linear systems by [16] to give MB-PETC. Advanced ET mechanisms (ETMs) were introduced which reduce the communication load in both SC and CA channels, and outperform the existing

ETMs. Closed-loop performance arbitrarily close to that of MB periodic time-triggered control (PTTC) setting was achieved. Also, a decentralized MB-PETC was provided suitable for large-scale systems. This work was extended by [56] by adding an approximate disturbance model which can further enhance communication savings in the presence of disturbances.

3.11. Event-based Model Predictive Control (MPC)

ET strategy for DT systems was proposed and analyzed in [11], where the plant was assumed ISS with respect to measurement errors, and the triggering condition was based on the norm of this error; the framework was used in MPC. The authors considered ET strategies for uncertain CT and DT nonlinear systems with additive disturbances under robust Nonlinear MPC (NMPC) in [12]. The updates of control law depended on the error between the actual and predicted trajectory of the system.

To deal with the systems with faster dynamics, the problem of robust networked static output feedback MPC design that stabilizes a class of linear uncertain systems was addressed in [57]. The methodology guaranteed cost, and gave a parameter-dependent quadratic stability (PDQS) which is based on Lyapunov–Krasovskii functional. The upper-bound on the delay was assumed to be bigger than the sampling time. Control design was based on sufficient robust stability condition formulated as a solution of bilinear matrix inequality BMI, which can be solved off-line.

A computationally efficient ET MPC scheme for CT nonlinear systems subject to bounded disturbances was given in [29]. First, an ET condition that depends on the error between the system state and its optimal prediction was designed, followed by the design of ET MPC algorithm that was built upon the triggering mechanism and the dual-mode approach. Feasibility and stability analysis were carried out in detail and sufficient conditions were thus presented. Specifically, it was shown that the proper selection of prediction horizon, and boundedness of the disturbance can guarantee feasibility of the ET MPC algorithm. Regarding stability, which is related to the prediction horizon, the disturbance bound, and the triggering level, it was shown that the state trajectory converges to a robust invariant set.

3.11.1. Discussion

Another result based on ET MPC was presented in [48] and is discussed in Section 3.5.2.

3.12. Event-based PID controller and actuator saturation

The effects of actuator saturation on the behavior of the ETC loop in terms of stability and communication were investigated by [23] using simulations. A static anti-windup mechanism was introduced to remedy the adverse effects on event-based control. Moreover, by means of LMIs stability regions were given, and a lower bound on the inter-event time was shown to exist. These results were also extended by considering a case where full-state information might not be available to the controller. The results were illustrated by simulations and experiments.

Remark 17. It was pointed out by the authors that alternate methods for deriving the stability regions can be pursued.

The design of an event-based PI control scheme for stable first-order processes was considered in [50] which aimed to diminish the oscillations around the set-point, and sticking effect. The conventional PI controller was replaced with PIDPLUS, a version of PI controller for NCSs which also deals with packet losses and time delays. Additionally, stability and performance analysis of the closed-loop system were provided. Simulations showed that the scheme ensures set-point tracking, disturbance rejection, and robustness against process delay while significantly reducing the number of transmissions.

Remark 18. Some potential future directions pointed out by the authors are:

(1) Consideration of multi dimensional systems,
(2) Investigation of the derivative part of PIDPLUS, and
(3) To further improve the trade-off between performance and event frequency.

3.12.1. Discussion

Although the proposed scheme in [50] is robust against the process delays, the stability analysis was not applicable to this case; this translates into an extension of the stability analysis to consider such delays.

3.13. Miscellaneous results

Efficient attentiveness property is exhibited by an event-based system when the length of inter-sampling interval increases monotonically as the sampled state approaches equilibrium. The authors in [64] established conditions on ET under which it can be guaranteed, in a computationally efficient manner, that the system possesses this property.

The suitability of resilient control for ET scheme was investigated in [27], and the authors suggested that such a control is achievable for at least transient faults. Resilient control system is the one which maintains state awareness while ensuring acceptable performance in response to disturbances. Required bit-rates were examined, and sufficient resilient bit-rates for nonlinear scalar systems with affine controls and disturbances were given. It was observed that the rates are independent of the initial states.

[44] presented the methodology to deal with bounded time delays in the NCSs. In order to gain apriori knowledge of the delays from a dynamic real-time behavior, dynamic priority exchange scheduling for bounding time-delays was used. The scheme also deals with faults which introduce a structural change in the dynamic model, and dynamic response due to real-time scheduling. The main contribution was to model the faults and delays, as perturbations considering nonlinear behavior through a fuzzy TKS approach in a co-design strategy.

The work of [22] deals with the problem of the ET fuzzy H_∞ control of a nonlinear NCS by using the deviation bounds of asynchronous normalized membership functions.

4. Self-triggered network control

In ST mechanism the feedback law computation is followed by determination of the next time instant to sample the state. This time instant is calculated on the basis of latest sampled state and plant dynamics; during these time instants, the system operates as open loop. This scheme mitigates the problems faced in the ET mechanism by eliminating the need to continually check plant's state against an event condition, and it can be regarded as a software based emulation of ET technique. In what follows, we present a survey of the results reported for ST scheme.

4.1. Stability

As mentioned above, the system operates in open-loop until the next update time, making stability and robustness primary issues. We present the literature which discusses the stability of ST schemes in terms of Input-to-State Stability (ISS) [35] and $\mathcal{L}_2$ stability [26,61,63].

4.1.1. ISS

The ISS for ST implementation was studied by [35] and it was shown to be exponentially uniformly ISS (U-ISS) with respect to the additive disturbances. Their main results are given in what follows. Consider an LTI system,

$$\dot{\xi}(t) = A\xi(t) + BK\xi(t_k) + \delta(t), \forall t \in [t_k, t_k + \Omega(\xi(t_k))), \tag{26}$$

where $\xi \in \mathbb{R}^m, K$ is the controller gain rendering the closed-loop system exponentially stable, and $\delta(t)$ is the additive disturbance, with ST implementation $\Omega : \mathbb{R}^m \to \mathbb{R}^+, \Omega(\xi(t_k)) = \tau_k$, determined by the policy,

$$\begin{aligned}
&\Omega(\xi(t_k)) := \max\{t_k + t_{min}, t_k + \tilde{n}_k\Delta\},\\
&\tilde{n}_k := \max\{s \leqslant N_{max} | \tilde{h}(n, \xi(t_k)) \leqslant 0, \forall n \in [0, s]\},\\
&\tilde{h}(n, x_k) := |\sqrt{P}R(n)x_k| - V(x_k)e^{-\lambda(n\Delta)},
\end{aligned} \tag{27}$$

where t_{min} is the minimum time between updates, Δ is the time interval to check triggering condition, N_{max} is the ratio of maximum inter-update time t_{max}, and λ is the decay rate of the Lyapunov function $V(x)$ with a positive definite symmetric matrix P and $R(n)$ is given as,

$$\begin{aligned}
&R(n) := A_d^n + \sum_{i=0}^{n-1} A_d^i B_d K,\\
&A_d := e^{A\Delta},\\
&B_d := \int_0^\Delta e^{A(\Delta-\tau)} B d\tau.
\end{aligned}$$

The parameters λ, Δ and N_{max} depend on the choice of designer, and t_{min} depends on the value of λ.

Theorem 5. *The system given by (26) with ST policy (27) is exponentially ISS.* $\square$

Remark 19. The authors indicated to apply similar ideas to non-linear systems (via approximate models) as future research.

4.1.2. $\mathcal{L}_2$ stability

Early co-design methodologies viewed the selection of inter-event time as an optimization problem. However, the cost functions (penalizing control performance) are rarely monotonic with respect to the inter-event time, which makes hard to find the optimal sampling time. Another approach based on Lyapunov techniques was used by [26], whereby the sampling periods were selected to ensure stability and performance in the presence of disturbance by adjusting the induced $\mathcal{L}_2$ gain. A ST real-time system implementing full information $\mathcal{H}_\infty$ controller along with a task scheduler was presented. Sampling times were utilized by scheduler to determine the actual release times.

We now present the performance results of a ST feedback control system due to [26]. Consider a real-time system with N plants controlled by a single processor with N *tasks*, where a task refers to three functions combined, namely: state sampling, control law computation, and control application using ZOH. The i-th plant is given as,

$$\begin{aligned}\dot{x}_i(t) &= A_i x_i(t) + B_{1i}u_i(t) + B_{2i}w_i(t),\\ x_i(0) &= x_{i0},\end{aligned} \tag{28}$$

where $i \in \{1,\ldots,N\}, w_i(t)$ is a bounded disturbance, and $u_i(t)$ is the control input computed by i-th task. Each task is associated with release times, $\{r_i[j]\}_{j=1}^{\infty}$, with $r_i[j]$ being the time when j-th job of i-th task is ready to be executed. The period for j-th job is given by,

$$T_i[j] = r_i[j+1] - r_i[j].$$

The control law $u_i(t)$ is of the form,

$$u_i(t) = -k^T x(r_i[j]). \tag{29}$$

Following theorem gives the main result.

Theorem 6. *Let G denote the sampled-data control system given by (28) and (29) with the control gain $k^T = -B_1^T P$, where P is a positive symmetric matrix that satisfies following ARE for some $\gamma > 0$,*

$$A^T P + PA + I - P\left(B_1B_1^T - \frac{1}{\gamma^2}B_2B_2^T\right)P = 0. \tag{30}$$

Let x_r denote system's state at release time $r[j]$. If the state $x(t)$ satisfies

$$\begin{bmatrix} x(t) \\ x_r \end{bmatrix}^T \begin{bmatrix} -I + PB_1B_1^T P & -PB_1B_1^T P \\ -PB_1B_1^T P & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x_r \end{bmatrix} \leqslant -\|x(t)\|^2, \tag{31}$$

for all $t \in [r[j], r[j+1])$ and $j = 1,\ldots,\infty$, then the induced $\mathcal{L}_2$ gain of G is less than γ. □

Authors in [26] extended their work in [61] by deriving the bounds on a task's sampling period and deadline, to quantify how robust control system's performance will be to the variations in these parameters. They developed inequality constraints on control task's period and deadline, whose satisfaction ensured that the system's induced $\mathcal{L}_2$ gain lies below a specified performance threshold. The results apply to LTI systems driven by bounded external disturbances.

Remark 20. The authors pointed out that the implementation of STC over WSAN can be pursued as an interesting future direction because of the inability of such networks to provide deterministic guarantees on message delivery.

The assumption on bounds of external disturbances in [61] was relaxed by [63], and it was shown that the sampling periods are always greater than a positive constant and larger than those generated in [61]. Moreover, the scheme is robust against external disturbances.

4.1.3. Discussion

The ISS results given here [35] do not consider real-time network issues such as packet loss and delay, which makes the stability results conservative for real-time applications.

Preliminary results for $\mathcal{L}_2$ stability in [26] show great robustness to scheduling delays induced by the real-time schedulers.

4.2. Self-triggered control of linear systems

An approach to design STC for linear systems was presented in [9] by exploiting the properties of universal formula for event-based control of nonlinear systems. The proposed methodology was shown to give better results than the existing ST approaches.

Remark 21. The authors indicated to extend the work to nonlinear systems and to test it on a real-time application.

Previous results on the state feedback stabilization were extended by [1] to the case of dynamic output feedback using DT observer. Global asymptotic stability is guaranteed for some observability condition.

Remark 22. The authors highlighted following future extensions of their work:

(1) Consideration of different scheduling methods in the observer based approach, and
(2) Consideration of general plant interconnections.

We now present stability results due to [1]. The LTI system is given as,

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t),\\ y(t) &= Cx(t),\end{aligned} \tag{32}$$

where $x \in \mathbb{R}^n$ with initial state x_0, the input $u \in \mathbb{R}^m$ and plant output $y \in \mathbb{R}^p$. The plant is assumed to be controllable and observable. The event-scheduler simulates a copy of plant dynamics (32) with control input $u(t) = Kx_k$, given as,

$$\dot{\xi}_x(t) = A\xi_x(t) + BKx, \quad \xi_x(0) = x, \tag{33}$$

where $\xi \in \mathbb{R}^n$. The ST scheme uses Lyapunov function approach to compute the triggering time. Let $V(x) = \sqrt{x^T P x}$ with positive definite symmetric P, be the Lyapunov function for (32), then its decay rate λ is selected by the designer such that,

$$V(x) \leqslant V(x_0)e^{-\lambda\tau}. \tag{34}$$

Let the function $h(\tau, x)$ be defined as,

$$h(\tau, x) = V(\xi_x(\tau)) - V(x)e^{-\lambda\tau}. \tag{35}$$

This function is used to get sampling times as,

$$\tau_{gridded}(x) = \Delta \max\{1 \leqslant d_2 \leqslant M : h(d_1\Delta, x) \leqslant 0, \quad \forall 1 \leqslant d_1 \leqslant d_2 \leqslant M\}, \tag{36}$$

where $0 \leqslant \Delta \leqslant \tau_{min}$, with τ_{min} being the lower bound on triggering time, and $M \in \mathbb{N}$ are design parameters. In order to estimate full state, observer is used. The control is computed as,

$$\begin{aligned} u(t) &= K\hat{x}_k, \quad t \in [t_k, t_{k+1}), \\ t_{k+1} &= t_k + \tau_{gridded}(\hat{x}_k). \end{aligned} \tag{37}$$

Let $\tilde{x}_k = x_k - \hat{x}_k$ for $k \geqslant 0$ be the observation error then the following theorem gives the stability results.

Theorem 7. *ST controller (37) renders plant (32) exponentially ISS with respect to observation errors, i.e., there exist positive constants σ and γ such that,*

$$\|x(t)\| \leqslant \sigma e^{-\lambda(t-t_0)}\|x_0\| + \gamma \max_{j\in\{0,1,\ldots,k\}}\|\tilde{x}_j\|, \tag{38}$$

for all $t \geqslant t_0$, where $k = \max\{p \geqslant 0 : t_p \leqslant t\}$. □

4.3. Self-triggered control of nonlinear systems

The first result for STC of nonlinear systems was presented by [3] for two classes of nonlinear systems, namely: state-dependent homogenous, and polynomial systems. Conditions defining next task execution times were shown to depend upon the dynamics of plant, desired performance, and the current state. Additionally, the authors provided an analysis to quantify the trade-off between performance requirement and communication resource usage. The proposed scheme was applied to the models of jet engine compressor and rigid body; the simulations showed that the inter-execution times due to ST scheme are an order of a magnitude larger than the periodic implementation, while giving the same performance. Also, the scheme is robust against disturbance and sensor noise.

This approach exploited the geometry of systems to scale the execution times along the surface in state-space, termed as *manifold*. As a result, a two-step approach was used to get the execution times, whereby the first step gives a rough estimate of the lower bound of inter-execution time, which is valid on a ball around the origin, and second step exactly scales this time to whole operating region to describe its evolution. In order to mitigate the conservatism introduced due to in-exact lower bound, the authors gave a detailed discussion in [4] on using *isochronous manifolds* (which are surfaces in state-space with states for which the execution times remain constant) that replace the ball used in the first step, to give an exact lower bound. This amalgam of both ST techniques was shown to outperform the existing ST methodologies. Moreover, the main results can be applied to any smooth control system by homogenizing it using the presented technique.

A limitation of [3,4] came from the approximation of isochronous manifolds, since it is impossible to obtain them in closed-form, in general. Also, the method to compute the manifold is not always applicable. In addition to this, the authors did not consider the effects of external disturbances and time delays. In order to tackle these issues, [51] proposed a simple ST sampler for perturbed nonlinear systems, subject to bounded external disturbance and small time delays, which ensured UUB of the trajectories. Moreover, in order to reduce the conservativeness, techniques based on disturbance observers were proposed. The methodology was validated through simulations.

The stability results for ST sampler due to [51] are briefly presented here. Consider a perturbed system given by,

$$\dot{x} = f(x, u, d), \tag{39}$$

where $x \in \mathcal{D}_x \subseteq R^n$ is the state, input is given as $u \in \mathcal{D}_u \subseteq R^p$, and disturbance $d \in \mathcal{D}_d \subseteq R^d$ is bounded as $\|d\| \leqslant \overline{d}$. Assume that there exists a differentiable state feedback law $u(t) = \kappa(x)$ with $\kappa : \mathcal{D}_x \to \mathcal{D}_u$, such that origin of the unperturbed system, $\dot{x} = f(x, \kappa(x), 0)$ is a unique locally asymptotically stable equilibrium point in $\mathcal{D}_x$. Furthermore, assume that the function $f(x, \kappa(x), d)$ is continuous over $\mathcal{D}_x \times \mathcal{D}_u \times \mathcal{D}_d$ with Lipschitz continuous derivatives. Let the Lipschitz constants for f and κ with respect to u and x be $L_{f,u}$ and $L_{\kappa,x}$, respectively.

Consider now the sampled data version of system (39),

$$\dot{x} = f(x, \kappa(x_k), d), \tag{40}$$

where $u(t) = \kappa(x_k)$ for $t \in [t_k, t_{k+1})$. Let $g(t)$ be defined as,

$$g(t) := f(x, \kappa(x_k), d) - f(x, \kappa(x), d), \tag{41}$$

and this function is bounded by some $\delta > 0$ such that, $g(t) \leqslant \delta$. A ST sampler to ensure GUUB of the system (40) is given by the following theorem:

Theorem 8. *For sampled data system (40) along with the assumptions stated above, ST sampler,*

$$t_{k+1} = t_k + \frac{1}{2L} \ln\left(1 + \frac{2\delta}{\|f(x^*, \kappa(x_k), d^*)\|}\right), \tag{42}$$

ensures GUUB of the closed-loop system, with $L = L_{f,u} L_{\kappa,x}$ *and* $(x^*, d^*) := \text{argmax}_{(y_1, y_2) \in \mathbb{R}^n \times \mathcal{D}_d} \|f(y_1, \kappa(x_k), y_2)\|$. □

Small-gain approach was used in [53] to develop STC, whereby the violation of small-gain condition marks a sampling event and computation of fresh control law, yielding a stable nonlinear system. Additionally, the approach does not require construction of a Lyapunov function. The proposed scheme was successfully applied to a trajectory tracking problem.

Remark 23. As an extension of their work, the authors highlighted the use of MB estimation of control and output signals instead of the conventional ZOH approximation to further increase the inter-sample time.

4.3.1. Discussion

The results presented in [3] take the time quantization aspect of real-time implementation into account and can be extended to time-delay case.

4.4. Minimum attention and anytime attention control

Two attention aware control schemes in the domain of NCSs are Minimum Attention, and Anytime Attention control. The former refers to the scenario whereby the control loop is closed only when necessary while satisfying certain performance requirements, for later, the system is allowed to run in open-loop for a *pre-scheduled* amount of time until the next control input is computed while fulfilling some performance requirements. Previous works on these control schemes had their limitations like, large computational load even for linear systems, and the assumption of availability of smart actuators which are rarely available. Other drawback reported for non-linear systems was that the input might drive the system into a state where more executions are needed to stabilize it making the method computationally intensive [64].

This problem was addressed by [8] using the ∞-norm-based extended control Lyapunov function (CLF), which allowed the minimum attention control problem to be formulated as a linear program solved efficiently online. Additionally, they considered only a finite number of possible inter-execution times.

4.4.1. Discussion

A comparison was presented between minimum attention control problem and STC by [8]. The difference lies in the fact that, the ST strategy is *emulation-based* which requires a two-step approach to design the controller i.e., firstly a feedback controller is designed assuming ideal communication and then the triggering mechanism is designed. This approach seldom gives an optimal solution. As compared with this, minimum attention control considers the control and triggering mechanism design simultaneously, and is more likely to yield a close-to optimal design. It was shown that minimum attention control outperformed the ST control scheme.

4.5. Miscellaneous results

The results for ETNC for DT systems were also extended to ST scheme by [11]. A simulation study was done in [52] on first order processes under STC framework. The proposed methodology uses a PI controller and disturbance observer, and considers process time-delays larger than the inter-sampling times.

For the first time, ST scheme for nonlinear stochastic systems with additive noise was considered by [2]. Strictly positive inter-execution times were observed that guarantee p-moment stability of the process.

Remark 24. As possible future extensions of this work, the authors pointed out to:

(1) Obtain a less conservative sampling rule by following the analysis for deterministic systems to further increase the inter-execution times,
(2) Study the robustness against task delay, and
(3) To apply the methodology to situations with limited control updates or state sampling.

Design of ST controller with the switched system approach was presented in [49] to further improve the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ performance. The problem was solved by first considering a linear quadratic problem for periodic sampling case, then using it for the development of $\mathcal{H}_2$ and $\mathcal{H}_\infty$ performance indices. The proposed methodology was validated on numerical examples.

5. Comparison

In this section we compare ET and ST control by re-simulating the work presented in [61], which compared their proposed ST and ET schemes. Due to space constraint, we briefly describe the methodologies. The LTI system considered is given as,

$$\begin{aligned}\dot{x}(t) &= Ax(t) + B_1u(t) + B_2w(t),\\ x(0) &= x_0,\end{aligned} \tag{43}$$

where $x_0 \in \mathbb{R}^n$ is non-zero initial state, $u : [0,\infty) \to \mathbb{R}^m$ is control input and $w : [0,\infty) \to \mathbb{R}^l$ is exogenous disturbance in $\mathcal{L}_2$ space. The controller used for both ET and ST schemes is full-information $\mathcal{H}_\infty$ controller which is designed assuming a symmetric positive semi-definite matrix P satisfying the following ARE,

$$0 = PA + A^TP - Q + R, \tag{44}$$

where

$$\begin{aligned}Q &= PB_1B_1^TP,\\ R &= I + \frac{1}{\gamma^2}PB_2B_2^TP,\end{aligned} \tag{45}$$

for some real $\gamma > 0$. This renders the closed-loop system,

$$\begin{aligned}\dot{x}(t) &= Ax(t) + B_1u(t),\\ u(t) &= -B_1^TPx(t),\end{aligned} \tag{46}$$

asymptotically stable. It can be denoted as $A_{cl} = A + B_1K$ with $K = -B_1^TP$. The closed-loop system is finite-gain $\mathcal{L}_2$ stable from the disturbance w to $(x^T, u^T)^T$ with an induced gain less than γ.

A sampled data implementation of the closed-loop,

$$\begin{aligned}\dot{x}(t) &= Ax(t) + B_1u(t) + B_2w(t),\\ u(t) &= -B_1^TPx(t),\end{aligned} \tag{47}$$

is considered, whereby the control computation is done by a computer task. Each task is characterized by release times r_k and finish times f_k with $k = 0, \ldots, \infty$, illustrated in the timing diagram in Fig. 2. The control signal is kept constant by ZOH until the next finishing time and the state trajectories are continuous, giving the sampled data system as,

$$\begin{aligned}\dot{x}(t) &= Ax(t) + B_1u(t) + B_2w(t),\\ u(t) &= -B_1^TPx(r_k),\end{aligned} \tag{48}$$

for $t \in [f_k, f_{k+1})$.

Let $e_k(t) = x(t) - x(r_k)$ be the error representing the difference between current and last release time state $x(r_0) = x_0$, and Q be the real matrix satisfying (45). For any $\beta \in [0,1)$, let

$$\begin{aligned}M &= (1-\beta^2)I + Q,\\ N &= \frac{1}{2}(1-\beta^2)I + Q.\end{aligned} \tag{49}$$

Now, let

$$z_k(t) = \sqrt{(1-\beta^2)I + Q}\,e_k(t) = \sqrt{M}e_k(t), \tag{50}$$

and

$$\rho(x) = \sqrt{x^TNx}, \tag{51}$$

then the ET scheme is given by the following theorem.

Theorem 9. *Consider sampled data system (48) satisfying the assumption that, for a real constant $W > 0$, $\|w(t)\|_2 \leqslant W\|x(t)\|_2$ for all $t \geqslant 0$. Assume that M has full rank and for some $\delta \in [0,1)$ the release time sequence $\{r_k\}_{k=0}^{\infty}$ satisfies,*

$$\|z(r_{k+1})\|_2 = \delta\rho(x(r_k)), \tag{52}$$

where $f_k = r_k \; \forall \; k = 0, \ldots, \infty$. Then sampled data system is finite-gain $\mathcal{L}_2$ stable from w to x with an induced gain less than γ/β. □

Let,

$$\alpha = \|\sqrt{M}A\sqrt{M}^{-1}\| + W\|\sqrt{M}B_2\|\|\sqrt{M}^{-1}\|, \tag{53}$$

and $\mu_0 : \mathbb{R}^n \to \mathbb{R}$ is a real-valued function given as,

$$\mu_0(x(r_k)) = \|\sqrt{M}A_{cl}x(r_k)\|_2 + W\|\sqrt{M}B_2\|\|x(r_k)\|_2. \tag{54}$$

For some $\epsilon \in (0,1)$, let $\phi : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}$ and $\mu_1 : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ be real valued functions defined as,

$$\begin{aligned}\mu_1(x(r_k), x(r_{k-1})) = W\|\sqrt{M}B_2\|\|x(r_k)\|_2 + \|\sqrt{M}(Ax(r_k) - B_1B_1^TPx(r_{k-1}))\|_2, \quad \phi(x(r_k), x(r_{k-1}); t - r_k) \\ = \frac{\mu_1(x(r_k), x(r_{k-1}))}{\alpha}\left(e^{\alpha(t-r_k)} - 1\right).\end{aligned} \tag{55}$$

For $0 \leqslant D_k = f_k - r_k$ and some $\eta \in (\epsilon, 1]$, let $L_2 : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R} \times (0,1] \to \mathbb{R}$ be defined as,

$$L_2(x(r_k), x(r_{k-1}); D_k, \eta) = \frac{1}{\alpha}\ln\left(1 + \alpha\frac{\eta\rho(x(r_k)) - \phi(x(r_k), x(r_{k-1}); D_k)}{\mu_0(x(r_k)) + \alpha\phi(x(r_k), x(r_{k-1}); D_k)}\right). \tag{56}$$

Furthermore, let $\xi : \mathbb{R}^n \times (0,1) \times (0,1) \to \mathbb{R}$ be a real valued function defined as,

$$\xi(x(r_{k-1}); \epsilon, \delta) = \frac{1}{\alpha}\ln\left(1 + \frac{\epsilon\delta\rho(x(r_{k-1}))}{\delta\rho(x(r_{k-1})) + \frac{\mu_0(x(r_{k-1}))}{\alpha}}\right), \tag{57}$$

then the proposed ST scheme is given by the following theorem.

Theorem 10. *Consider sampled data system (48) satisfying the bounded disturbance assumption that, for a real constant $W > 0, \|w(t)\|_2 \leqslant W\|x(t)\|_2 \; \forall \; t \geqslant 0$. Assume that M has full rank, for some $\epsilon \in (0,1)$ and $\delta \in (\epsilon, 1)$, we assume that*

(1) *The initial release and finish times satisfy, $r_{-1} = r_0 = f_0 = 0$.*
(2) *For any non-negative integer k, the release times are generated by,*

$$r_{k+1} = f_k + L_2(x(r_k), x(r_{k-1}); D_k, \delta), \tag{58}$$

where L_2 is given in (56), and the finish times satisfy,

$$r_{k+1} \leqslant f_{k+1} \leqslant r_{k+1} + \xi(x(r_k); \epsilon, \delta), \tag{59}$$

with ξ defined in (57).

Then the sampled data system is said to be finite-gain $\mathcal{L}_2$ stable from w to x with an induced gain less than γ/β. □

5.1. Simulation

The plant considered is an inverted pendulum on top of a moving cart with states $x = [y \quad \dot{y} \quad \theta \quad \dot{\theta}]^T$, where y and θ denote cart's position and pendulum bob's angle, respectively. The system matrices are given as,

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{mg}{M} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{g}{l} & 0 \end{bmatrix}; \quad B_1 = \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ -\frac{1}{Ml} \end{bmatrix}; \quad B_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \tag{60}$$

where m is bob's mass, M denotes cart's mass, l is the length of the pendulum, and g is gravitational acceleration. The values taken for these parameters are $m = 1, M = 10, l = 3$ and $g = 10$. The initial state for the system is $x_0 = [0.98 \quad 0 \quad 0.2 \quad 0]^T$. The $\mathcal{H}_\infty$ controller is designed using MATLAB with $\gamma = 200$ to obtain the vector K as,

$$K = [-2 \quad -12 \quad -378 \quad -210]. \tag{61}$$

5.2. Results

The ET and ST schemes, given by Theorems 9 and 10, were implemented using Simulink on the given system, with $\epsilon = 0$ and $\delta = 1$, i.e., without delay. The state errors resulting from both schemes are compared using the normalized state error (NSE) given as,

$$E(t,x) = \frac{|\sqrt{V(x(t))} - \sqrt{V(x_c(t))}|}{\sqrt{V(x_c(t))}}, \tag{62}$$

where $x(t)$ denotes the state of ET or ST controlled system, $x_c(t)$ is the CT system state and $V(x)$ represents the Lyapunov function for the system i.e., $V(x) = x^T Px$. Fig. 3 shows NSE for both schemes with $w(t) = 0$. It can be seen that the error for ET controlled system is slightly greater than that for ST scheme. Fig. 4 shows similar results for the case when the system was subject to a bounded disturbance with $W = 0.01$, given as,

$$\begin{aligned} w(t) &= \mathrm{sgn}(\sin t), \quad 0 \leqslant t < 10, \\ &= 0, \quad \text{otherwise}. \end{aligned} \tag{63}$$

Figs. 5 and 6 show the periods generated by ET and ST schemes, respectively, for the case without disturbance. ET generated periods which range from 0.039 to 0.9410 s and ST gave periods ranging from 0.065 to 0.106 s. These results conform

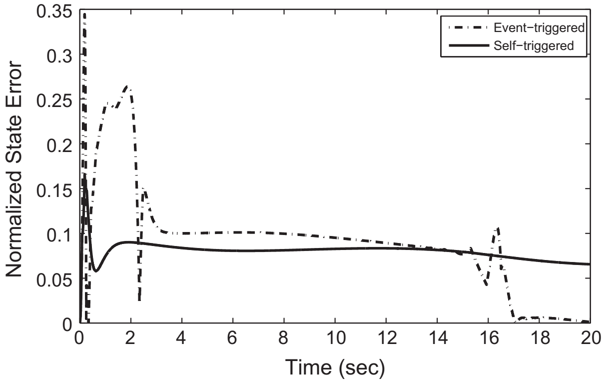

Fig. 3. Normalized state errors for event- and self-triggered control schemes for $w(t) = 0, \delta = 1$ and $\epsilon = 0$.

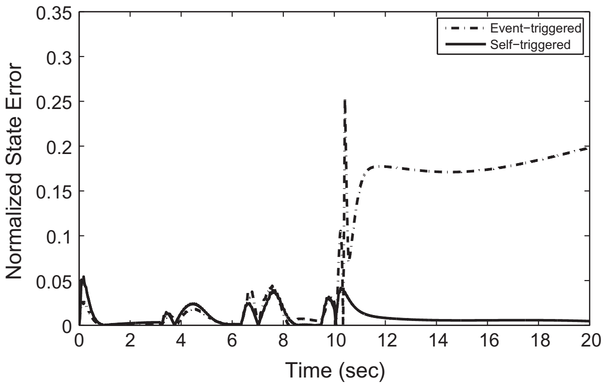

Fig. 4. Normalized state errors for event- and self-triggered control schemes for $w(t)$ as given in (63), $\delta = 1$ and $\epsilon = 0$.

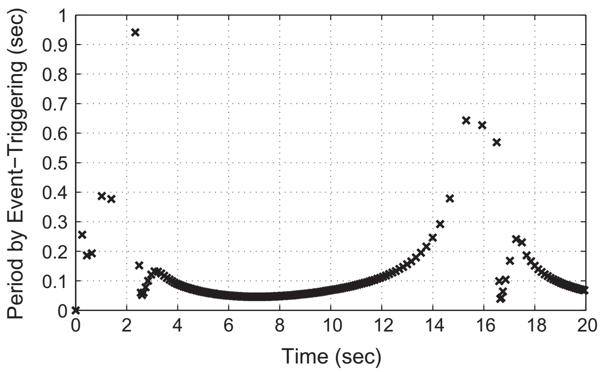

Fig. 5. Sampling period versus time for event-triggered control scheme for $w(t) = 0, \delta = 1$ and $\epsilon = 0$.

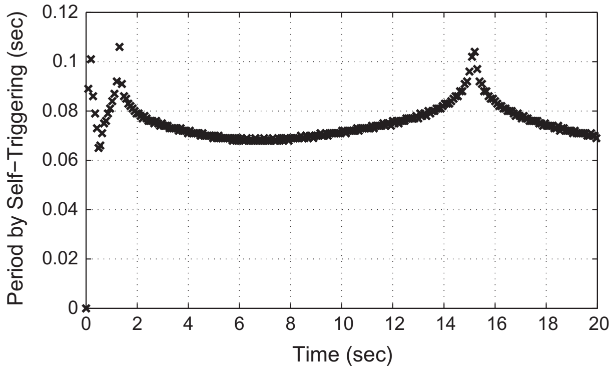

Fig. 6. Sampling period versus time for self-triggered control scheme for $w(t) = 0, \delta = 1$ and $\epsilon = 0$.

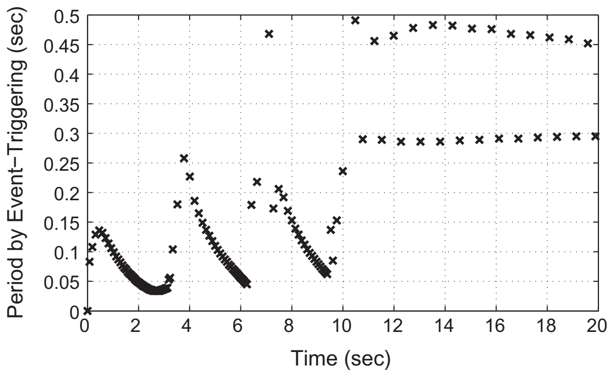

Fig. 7. Sampling period versus time for event-triggered control scheme for $w(t)$ as given in (63), $\delta = 1$ and $\epsilon = 0$.

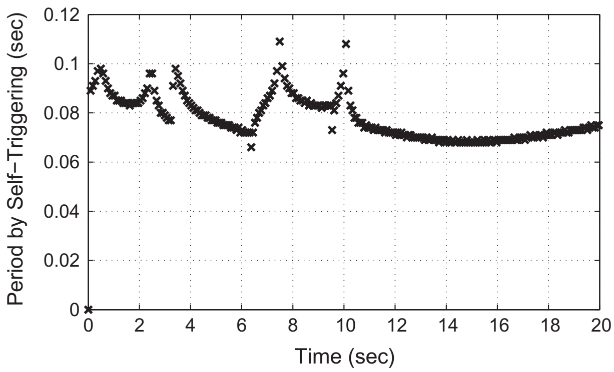

Fig. 8. Sampling period versus time for self-triggered control scheme for $w(t)$ as given in (63), $\delta = 1$ and $\epsilon = 0$.

with those reported in [61], i.e., the periods generated by ET methodology are larger as compared with the ST scheme due to the conservative nature of ST update times. Also, for the case with disturbance, ET scheme (Fig. 7) generated periods that are larger than ST scheme (Fig. 8); ET periods ranged between 0.034 and 0.4910 s while that for ST were between 0.066 and 0.1 s. It can also be observed that the periods get smaller when there is a disturbance effecting the system, this ensures the required performance of the overall system showing that both the schemes are robust against changes in the disturbance.

6. Conclusion

During the last decade, the literature on ET and ST control strategies has expanded to develop systems theory on the subject due to a significant cost saving that these schemes offer as compared to the conventional periodic triggering control methodology. In this survey, we tried to cover most of the relevant theoretical aspects to give a big picture of the progress, in an effort to help organize the scattered results and also ease the search of potential research avenues.

Most of the works cited here, particularly in the context of ET-based output feedback control and estimation consider a delay free communication channel, translating into the consideration of real-time network conditions as a possible area for future research.

In general, the area in which the subject needs further efforts is the development of comprehensive co-design methodologies while considering practical aspects of actual implementation scenarios. In connection to this, the experimental validation of these results would make a significant contribution, and a step towards replacing the periodic schemes in the industry.

Remark 25. Due to space limitations and Associate Editor's suggestion, some of the publications have been excluded. However, full data-base can be provided to the interested readers upon request.

Acknowledgments

The authors would like to thank the AE and reviewers for their constructive feedback and valuable comments on our submission. This work is supported by the Deanship of Scientific Research (DSR) at KFUPM through research group project RG-1316-1.

Appendix A

See Table I.

Table I
List of publications which consider time-delay.

Ref.	Comments
STABILITY OF ET SYSTEMS	
[37]	Bounded time-varying delays in CA channel. Incorporates the delay in system definition and co-design framework
[19]	Time-delay in SC and CA channels. Maximizes the inter-transmission times which reduces the effect of the delays
[31]	Uses a congestion avoidance module, which schedules the signals, to lessen the effect of network induced delay
SCHEDULING AND EVENT DESIGN	
[65]	Delays in embedded systems
CO-DESIGN OF ET SYSTEMS	
[73]	Transmission delays. Uses a delay system model which models delays and the event-driven system
[43]	Co-design algorithm gives the maximum allowable communication delay bound (MADB) and the maximum allowable number of successive packet losses (MANSPL)
[5]	The model considers retransmission of unsuccessful trials, and it interprets them as a delay associated with these retransmissions
[40]	Result is valid in the presence of delays and dropouts if instantaneous error-free acknowledgment channel exists
[41]	A TCP-like communication system is assumed, i.e. the communication system is equipped with an acknowledgment channel that informs the event-trigger, whether a transmission has been successful. The acknowledgment channel is error-free
OUTPUT FEEDBACK BASED ETC AND EVENT BASED ESTIMATION	
[72]	Time-delay in SC and CA channels. Achieves finite-gain $\mathcal{L}_2$ stability in the presence of arbitrary constant or time-varying delays, with bounded jitters
[74]	The analysis accounts for bounded time-delays
[47]	The analysis can be extended to include packet delays, dropouts and disorder
ET QUANTIZED SYSTEMS	
[70]	ET-based CA network. Uses predictive-control methodology to deal with packet delays, dropouts and disorders
[20]	Considers delay model to take into account ET mechanism, delays, and quantization in a unified framework
[30]	Analyzes asymptotically stabilizing bit-rates for maximum acceptable delays
ETC OF DECENTRALIZED AND DISTRIBUTED SYSTEMS	
[36]	Briefly considers bounded delays
[67]	Provides global tradeoff condition between all transmission periods and all kinds of allowable delays that ensures asymptotic stability
[59]	Predicts maximal allowable transmission delay of a subsystem's broadcast based on the local information
[66]	Proves that for non-zero transmission delays less than the corresponding deadline, the NCS is asymptotically stable
[15]	Proposes two communication protocols which guarantee stable behavior in the presence of network imperfections
[13]	Considers time-varying network delays
EVENT-BASED MODEL PREDICTIVE CONTROL (MPC)	
[57]	Assumes that the upper-bound on the delay is bigger than the sampling time
EVENT-BASED PID CONTROLLER AND ACTUATOR SATURATION	
[50]	Uses PIDPLUS, a NCS version of PI controller that deals with packet losses and time delays
MISCELLANEOUS RESULTS FOR ET SYSTEMS	
[44]	Bounded-time delays. Gains apriori knowledge of the delays by using dynamic priority exchange scheduling
STABILITY OF ST SYSTEMS	
[26]	Robust against scheduling delays induced by the real-time schedulers
STC OF NONLINEAR SYSTEMS	
[51]	Small time-delays. Proposes a simple ST sampler for perturbed nonlinear systems
MISCELLANEOUS RESULTS FOR ST SYSTEMS	
[52]	Considers process time-delays larger than the inter-sampling times

References

[1] J. Almeida, C. Silvestre, A.M. Pascoal, Self-triggered output feedback control of linear plants, in: Proc. 2011 American Control Conf., San Francisco, CA., 2011, pp. 2831–2836.
[2] R.P. Anderson, D. Milutinović, D.V. Dimargonas, Self-triggered stabilization of continuous stochastic state-feedback controlled systems, in: Proc. 2013 European Control Conf. (ECC), Zürich, Switzerland, 2013, pp. 1151–1155.
[3] A. Anta, P. Tabuada, To Sample or not to sample: self-triggered control for nonlinear systems, IEEE Trans. Autom. Control 55 (9) (2010) 2030–2042.
[4] A. Anta, P. Tabuada, Exploiting isochrony in self-triggered control, IEEE Trans. Autom. Control 57 (4) (2012) 950–962.
[5] B. Demirel, V. Gupta, M. Johansson, On the trade-off between control performance and communication cost for event-triggered control over lossy networks, in: Proc. European Control Conf. (ECC), Zurich, Switzerland, 2013, pp. 1168–1174.
[6] C. De Persis, R. Sailer, F. Wirth, Parsimonious event-triggered distributed control: a Zeno free approach, Automatica 49 (7) (2013) 2116–2124.
[7] M.C.F. Donkers, W.P.M.H. Heemels, Output-based event-triggered control with guaranteed $\mathcal{L}_\infty$-gain and improved and decentralized event-triggering, IEEE Trans. Autom. Control 57 (6) (2012) 1362–1376.
[8] M.C.F. Donkers, P. Tabuada, W.P.M.H. Heemels, On the minimum attention control problem for linear systems: A linear programming approach, in: Proc. 50th IEEE Conf. on Decision and Control and European Control Conf. (CDC-ECC), Orlando, FL, 2011, pp. 4717–4722.
[9] S. Durand, J.F. Guerrero-Castellanos, R. Lozano-Leal, Self-triggered control for the stabilization of linear systems, in: Proc. 9th IEEE Int. Conf. on Elect. Eng., Computing Sci. and Automat. Control (CCE), 2012, pp. 1–6.
[10] S. Durand, J.F. Guerrero-Castellanos, N. Marchand, W.F. Guerrero-Sánchez, Event-based control of the inverted pendulum: swing up and stabilization, Control Eng. Appl. Informatics 15 (3) (2013) 96–104.
[11] A. Eqtami, D.V. Dimarogonas, K.J. Kyriakopoulos, Event-triggered control for discrete-time systems, in: Proc. of American Control Conf. ACC, 2010, pp. 4719–4724.
[12] A. Eqtami, D.V. Dimarogonas, K.J. Kyriakopoulos, Novel event-triggered strategies for Model Predictive Controllers, in: Proc. 50th IEEE Conf. on Decision and Control and European Control Conf. (CDC-ECC), 2011, pp. 3392–3397.
[13] E. Garcia, P. Antsaklis, Model-based event-triggered control for systems with quantization and time-varying network delays, IEEE Trans. Autom. control 58 (2) (2013) 422–434.
[14] M. Guinaldo, D.V. Dimarogonas, K.H. Johansson, J. Sánchez, S. Dormido, Distributed event-based control strategies for interconnected linear systems, Control Theory Appl. 7 (6) (2013) 877–886.
[15] M. Guinaldo, D. Lehmann, J. Sánchez, S. Dormido, K.H. Johansson, Distributed event-triggered control with network delays and packet losses, in: Proc. 2012 IEEE 51st Annu. Conf. on Decision and Control (CDC), 2012, pp. 1–6.
[16] W.P.M.H. Heemels, M.C.F. Donkers, Model-based periodic event-triggered control for linear systems, Automatica 49 (2013) 698–711.
[17] W.P.M.H. Heemels, M. Donkers, A. Teel, Periodic event-triggered control for linear systems, IEEE Trans. Autom. Control 58 (4) (2013) 847–861.
[18] W.P.M.H. Heemels, J.H. Sandee, P.P.J. Van Den Bosch, Analysis of event-driven controllers for linear systems, Int. J. Control 81 (4) (2008) 571–590.
[19] S. Hu, D. Yue, $\mathcal{L}_2$-gain analysis of event-triggered networked control systems: a discontinuous Lyapunov functional approach, Int. J. Robust Nonlinear Control (2012), http://dx.doi.org/10.1002/rnc.2815.
[20] S. Hu, D. Yue, Event-triggered control design of linear networked systems with quantizations, ISA Trans. 51 (1) (2012) 153–162.
[21] L. Jetto, V. Orsini, A new event-driven output-based discrete-time control for the sporadic MIMO tracking problem, Int. J. Robust Nonlinear Control (2012), http://dx.doi.org/10.1002/rnc.2921.
[22] X.C. Jia, X.-B. Chi, Q.L. Han, N.-N. Zheng, Event-triggered fuzzy $\mathcal{H}_\infty$ control for a class of nonlinear networked control systems using the deviation bounds of asynchronous normalized membership functions, Inf. Sci. 259 (2014) 100–117.
[23] G.A. Kiener, D. Lehmann, K.H. Johansson, Actuator saturation and anti-windup compensation in event-triggered control, Discrete Event Dyn. Syst. (2012), http://dx.doi.org/10.1007/s10626-012-0151-1.
[24] D. Lehmann, J. Lunze, Extension and experimental evaluation of an event-based state-feedback approach, Control Eng. Practice 19 (2) (2011) 101–112.
[25] M. Lemmon, Event-triggered feedback in control, estimation and optimization, in: Lecture Notes in Control and Information Sciences, Springer-Verlag London Ltd., 2010, pp. 293–358.
[26] M. Lemmon, T. Chantem, X.S. Hu, M. Zyskowski, On self-triggered full-information H-Infinity controllers, in: Hybrid Syst.: Computation and Control, Springer, Berlin Heidelberg, 2007, pp. 371–384.
[27] L. Li, B. Hu, M. Lemmon, Resilient event triggered systems with limited communication, in: Proc. 2012 IEEE 51st Annu. Conf. on Decision and Control (CDC), 2012, pp. 6577–6582.
[28] L. Li, M. Lemmon, Weakly coupled event triggered output feedback control in wireless networked control systems, in: Proc. 2011 49th Annu. Allerton Conf. on Commun., Control, and Computing, Allerton, 2011, pp. 572–579.
[29] H. Li, Y. Shi, Event-triggered robust model predictive control of continuous-time nonlinear systems, Automatica 50 (5) (2014) 1507–1513.
[30] L. Li, X. Wang, M. Lemmon, Stabilizing bit-rates in quantized event triggered control systems, in: Proc. 15th ACM Intl. Conf. on Hybrid Syst.: Computation and Control, 2012, pp. 245–254.
[31] Y. Lin, Q.L. Han, F. Yang, Event-triggered control for networked systems based on network dynamics, in: Proc. 2013 IEEE Int. Symp. on Ind. Electron. (ISIE), 2013, pp. 1–6.
[32] T. Liu, M. Cao, C. De Persis, J.M. Hendrickx, Distributed event-triggered control for synchronization of dynamical networks with estimators, Estimation Control Networked Syst. 4 (1) (2013) 116–121.
[33] J. Lunze, D. Lehmann, A state-feedback approach to event-based control, Automatica 46 (1) (2010) 211–215.
[34] N. Marchand, S. Durand, J.F.G. Castellanos, A general formula for event-based stabilization of nonlinear systems, IEEE Trans. Autom. Control 58 (5) (2013) 1332–1337.
[35] M. Mazo Jr., P. Tabuada, Input-to-state stability of self-triggered control systems, in: Proc. Joint 48th IEEE Conf. on Decision and Control and 28th Chinese Control Conf., Shanghai, P.R. China, 2009, pp. 928–933.
[36] M. Mazo, P. Tabuada, Decentralized event-triggered control over wireless sensor/actuator networks, IEEE Trans. Automat. Control 56 (10) (2011) 2456–2461.
[37] X. Meng, T. Chen, Event-based stabilization over networks with transmission delays, Control Sci. Eng. (2012), http://dx.doi.org/10.1155/2012/212035.
[38] A. Molin, S. Hirche, Optimal event-triggered control under costly observations, in: Proc. 19th Int. Symp. on Math. Theory of Networks and Syst, Budapest, Hungary, 2010, pp. 2203–2208.
[39] A. Molin, S. Hirche, Adaptive event-triggered control over a shared network, in: Proc. 2012 IEEE 51st Annu. Conf. on Decision and Control (CDC), Mauii, Hawaii, 2012, pp. 6591–6596.
[40] A. Molin, S. Hirche, On the optimality of certainty equivalence for event-triggered control systems, IEEE Trans. Autom. Control 58 (2) (2013) 470–474.
[41] A. Molin, S. Hirche, Suboptimal event-triggered control for networked control systems, ZAMM-J. of Appl. Math. and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik, 2013, doi: 10.1002/zamm.201100139.
[42] A. Molin, S. Hirche, A bi-level approach for the design of event-triggered control systems over a shared network, Discrete Event Dyn. Syst. 24 (2) (2014) 153–171.
[43] C. Peng, T.C. Yang, Event-triggered communication and control co-design for networked control systems, Automatica 49 (5) (2013) 1326–1332.
[44] H. B-. Peréz, A. B-. Peréz, J. O-. Arjona, Networked control systems design considering scheduling restricions and local faults using local state estimation, Int. J. Innovative Comput. Inf. Control 9 (8) (2013) 3225–3239.
[45] A. Sahoo, Adaptive state feedback event-triggered control of affine nonlinear discrete time systems, in: Proc. 7th Ann. ISC Graduate Research Symp. (ISC-GRS), Rolla, Missouri, 2013.

[46] A. Sahoo, H. Xu, S. Jagannathan, Adaptive event-triggered control of a uncertain linear discrete time system using measured input output data, in: Proc. 2013 American Control Conf. (ACC), Washington, DC, 2013, pp. 5672–5677.
[47] D. Shi, T. Chen, L. Shi, Event-based state estimation of linear dynamical systems: Communication rate analysis, in: Proc. American Control Conf. (ACC), Portland, Oregon, 2014, pp. 4665–4670.
[48] J. Sijs, M. Lazar, W.P.M.H. Heemels, On integration of event-based estimation and robust MPC in a feedback loop, in: Proc. 13th ACM Int. Conf. on Hybrid Syst.: Comput. and Control, 2010, pp. 31–40.
[49] M. Souza, G.S. Deaecto, J.C. Geromel, J. Daafouz, Self-triggered linear quadratic networked control, Optimal Control Appl. Methods (2013), http://dx.doi.org/10.1002/oca.2085.
[50] U. Tiberi, J. Araújo, K.H. Johansson, On event-based PI control of first-order processes, in Proc. IFAC Conf. on Advances in PID Control (PID'12), Brescia, Italy, 2012.
[51] U. Tiberi, K.H. Johansson, A simple self-triggered sampler for perturbed nonlinear systems, Nonlinear Anal. Hybrid Syst. 10 (2013) 126–140.
[52] U. Tiberi, C.F. Lindberg, A.J. Isaksson, Dead-band self-triggered PI control for processes with dead-time, Adv. PID Control 2 (1) (2012) 442–447.
[53] D. Tolić, R.G. Sanfelic, R. Fierro, Self-triggering in nonlinear systems: A small gain theorem approach, in: Proc. 20th Mediterranean Conf. on Control and Automation (MED), Barcelona, Spain, 2012, pp. 941–947.
[54] V. Vasyutynskyy, K. Kabitzsch, Event-based control: Overview and generic model, in: Proc. 8th IEEE Int. Workshop on Fact. Commun. Syst. (WFCS), 2010.
[55] M. Velasco, P. Martí, J.M. Fuertes, The self-triggered task model for real-time control systems, in: Proc. 24th IEEE Real-Time Syst. Symp. (RTSS03), vol. 384, 2003,.
[56] J.L.C. Verhaegh, T.M.P. Gommans, W.P.M.H. Heemels, Extension and evaluation of model-based periodic event-triggered control, in: Proc. 2013 European Control Conf. (ECC), Zürich, Switzerland, 2013, pp. 1138–1144.
[57] V. Veselý, D. Rosionová, T.N. Quang, Networked output feeddback robust predicive controller design, Int. J. Innovative Comput. Inf. Control 9 (10) (2013) 3941–3953.
[58] X. Wang, E. Kharisov, N. Hovakimyan, Real-time $\mathcal{L}_1$ adaptive control algorithm in uncertain networked control systems, IEEE Trans. Autom. Control (2011) (submitted for publication). Available online at: http://mechsenaira.web.engr.illinois.edu/wp-content/uploads/2011/12/event/_L1/_jnl/_event/_v1.pdf.
[59] X. Wang, M.D. Lemmon, Event-triggering in distributed networked systems with data dropouts and delays, in: Hybrid Syst.: Computation and Control, Springer, Berlin Heidelberg, 2009, pp. 366–380.
[60] P. Wang, M.D. Lemmon, Distributed network utility maximization using event-triggered barrier methods, in: Proc. European Control Conf. (ECC), 2009.
[61] X. Wang, M. Lemmon, Self-triggered feedback control systems with finite-gain $\mathcal{L}_2$ stability, IEEE Trans. Autom. Control 54 (3) (2009) 452–467.
[62] P. Wang, M.D. Lemmon, Event-triggered distributed optimization in sensor networks, in: Proc. Int. Conf. on Inform. Process. in Sensor Networks, 2009, pp. 49–60.
[63] X. Wang, M. Lemmon, Self-triggering under state independent disturbances, IEEE Trans. Autom. Control 55 (6) (2010) 1494–1500.
[64] X. Wang, M. Lemmon, Attentively efficient controllers for event-triggered feedback systems, in: Proc. 2011 50th IEEE Conf. on Decision and Control and European Control Conf. (CDC-ECC), Orlando, FL, 2011, pp. 4698–4703.
[65] X. Wang, M. Lemmon, On event design in event-triggered feedback systems, Automatica 47 (10) (2011) 2319–2322.
[66] X. Wang, M.D. Lemmon, Event-triggering in distributed networked control systems, IEEE Trans. Autom. Control 56 (3) (2011) 586–601.
[67] X. Wang, Y. Sun, N. Hovakimyan, Asynchronous task execution in networked control systems using decentralized event-triggering, Syst. Control Lett. 61 (9) (2012) 936–944.
[68] J. Weimer, J. Araújo, K.H. Johansson, Distributed event-triggered estimation in networked systems, in: Proc. of Conf. Anal. and Design of Hybrid Syst., 2012, pp. 178–185.
[69] J. Wu, Q. Jia, K.H. Johansson, L. Shi, Event-based sensor data scheduling: trade-off between communication rate and estimation quality, IEEE Trans. Autom. Control 58 (4) (2013) 1041–1046.
[70] R. Yang, G-. P. Liu, P. Shi, C. Thomas, M.V. Basin, Predictive output feedback control for networked control systems, IEEE Trans. Autom. Control 61 (1) (2014) 512–520.
[71] R. Yang, P. Shi, G-. P. Liu, H. Gao, Network-based feedback control for systems with mixed delays based on quantization and dropout compensation, Automatica 47 (12) (2011) 2805–2809.
[72] H. Yu, P.J. Antsaklis, Event-triggered output feedback control for networked control systems using passivity: Achieving $\mathcal{L}_2$ stability in the presence of communication delays and signal quantization, Automatica 49 (2013) 30–38.
[73] D. Yue, E. Tian, Q.L. Han, A delay system method for designing event-triggered controllers of networked control systems, IEEE Trans. Autom. Control 58 (2) (2013) 475–481.
[74] X.M. Zhang, Q.L. Han, Event-triggered dynamic output feedback control for networked control systems, IET Control Theory Appl. 8 (4) (2014) 226–234.