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Abstract: Some of the critical issues in networked control systems are optimisation of control, communication, and computational costs, time-delays, and coupling of control and triggering-condition. To address the optimisation of communication and control, an approach has been recently reported by Gommans *et al.* (2014), based on self-triggered (ST) linear quadratic (LQ) control. However, the optimisation of computational cost, coupling, and time-delays were not considered. With this motivation, a novel ST LQ methodology is proposed in this study. The features, merits, and demerits of this approach are compared against Gommans *et al.* (2014) and simulation results are presented for three case studies.

1 Introduction

With an escalating application of networked feedback loops, the demand to optimise and decouple control and communication, and to economise computational cost and energy consumption has also increased [1]. Aperiodic transmission schemes [event- and self-triggered (ET and ST)] have been the focus of many studies due to their energy and communication economy [2]. The complexity that these methodologies offer is the correlation between states of the participating control loops in networked control systems (NCSs), thus complicating the interaction between control and communication. In what follows, some highlighted works in the field of aperiodically triggered NCSs are discussed briefly.

As pointed out by Mahmoud and Memon [2], there are two methodologies to achieve simultaneous design of control and triggering mechanism in ET NCSs: cost function minimisation which penalises the inter-event times besides state and control variable [3–6], and Lyapunov approach [7–9]. Specifically, Demirel *et al.* [3] presented a theoretical framework to analyse the trade-off between control performance and communication cost for transmission over a lossy network. In [4], the certainty equivalence controller was reported to be optimal for ET control systems with some constraints. Two suboptimal design strategies for linear discrete-time (DT) stochastic systems with network imperfections were presented in [5]. To decouple communication from control in ET setting, Molin and Hirche [6] limited the usage of communication channel in terms of maximum allowable transmission rate. The authors used the solution of stochastic optimal control problem to determine optimal transmission rates.

The scheme proposed in [7] gave an algorithm for the *one-step* approach to co-design, as opposed to the previous methodologies which first design the controller with perfect signal transmission assumption and then consider ET scheme. ET $\mathcal{L}_2$ control for sampled-data systems was proposed in [8], which did not require any special hardware to monitor the plant state for event-condition. Recently, $\mathcal{L}_2$ controller design problem incorporating quantisation and time-varying delays was considered in [9]. The controller was designed with the help of a new model for NCSs including the state, network conditions, and ET communication strategy.

Another recent result reported by Zhang *et al.* [10] addressed the stabilisation problem for a class of non-linear NCSs with delays and quantisation through sampled-data control. For linear multi-agent systems, Zhang *et al.* [11] reported centralised and distributed observer-based ET schemes.

However, for ET the energy consumption is high due to continuous (or periodic) monitoring of the plant state, and it generally requires a dedicated hardware to check the event condition. Due to these reasons ET is not desirable, particularly for battery-powered sensor nodes in a wireless sensor network. ST mechanism introduced in [12] provides remedy to these problems by *predicting* the next update time on the basis of previously sampled state and plant dynamics.

ST methodology has been the focus of many works [13–23]. In [13], a full information ST $\mathcal{H}_\infty$ controller along with a task scheduler was presented. The authors extended this work in [14] by deriving the bounds on a task's sampling period and deadline, to quantify how robust control system's performance will be to the variations in these parameters. The results are applicable to linear time-invariant (LTI) systems driven by bounded external disturbances. The assumption on bounds on the disturbances was relaxed in [15], and it was shown that the sampling periods are always greater than a positive constant and larger than those generated in [14]. ST model predictive control was reported in [16], whereby the cost function, penalising sampling cost besides control, was solved off-line. Recently, Kögel and Findeisen [17] presented the design of practically implementable ST controllers for Lipschitz non-linear systems guaranteeing performance against disturbances.

Two control schemes which optimise the usage of communication channel are minimum attention and anytime attention controls [18–20]. The former refers to the scenario when the feedback is supplied only when necessary while satisfying certain performance requirements. For later, the system is allowed to run in open loop for a *pre-scheduled* amount of time until the next control input is computed while fulfilling some performance requirements. However, these ST controllers offered some limitations, such as being computationally intensive even for linear systems [18], offering suboptimal solutions, and high sampling frequency in some cases [19], and considering only a finite number of possible inter-execution times [20].

To the best of authors' knowledge, ST linear quadratic (LQ) control problem which also focuses on optimising communication channel usage, has been addressed only by the authors [21–23]. In particular, the approach of [21] involved the development of LQ problem for periodic case, and then using the result for $\mathcal{H}_2$ and $\mathcal{H}_\infty$

performance indices, yielding the solution of $\mathcal{H}_2$ and $\mathcal{H}_\infty$ optimal control problems. Lastly, in order to improve the performance, a ST controller was designed with switched system approach. These results were extended in [22] to consider dynamic output feedback. On the basis of the performance requirements fixed by the quadratic discounted cost, ST LQR was presented in [23]. The proposed scheme involved joint design of the regulator and triggering condition, and guaranteed a priori chosen performance levels by design. However, these works are deficient in,

- considering time-delays,
- presenting a decoupled design for control gain and triggering mechanism,
- optimising the computational resources besides control and communication, and
- designing ST sampler which causes lower sampling frequency when the plant is regulated and higher when in transient phase.

With this motivation, a novel ST sampling and control methodology is proposed. The network and computational delays are tackled by using asynchronous sampled-data system (ASDS) approach [24]; to the best of the authors' knowledge this approach is used for the first time for aperiodically triggered systems. Control and triggering method are decoupled by bounding the sampling time, and a new sampler is proposed which optimises the usage of communication channel. In addition, the methodology is compared against [23] on the basis of control cost incurred, communication bandwidth used, and computational requirements by presenting three simulation studies. To present a fair comparison, the time-delays are not considered in the simulation because a perfect communication channel is assumed in [23]. Nevertheless, the proposed methodology which incorporates the delays is presented for completeness.

Organisation: Section 2 defines the problem, and the reader will find a brief description of the methodology presented by Gommans *et al.* [23] in Section 3. The proposed scheme is then presented in Section 4. A comparison of both schemes is given in Section 5. Finally, Section 6 concludes the paper.

Notations: Sets of real numbers and integers are denoted by $\mathbb{R}$ and $\mathbb{Z}$, respectively. Set of strictly-positive real numbers is represented by $\mathbb{R}_+$ and that including zero is denoted as $\mathbb{R}_{\{0,+\}}$. The sets of positive-definite and semi-definite integers are represented as $\mathbb{Z}_+$ and $\mathbb{Z}_{\{0,+\}}$, respectively. The expected value is represented as $\mathbb{E}[.]$.

2 Problem definition

A class of LQ ST control systems is considered, associated with a continuous-time (CT) LTI plant given as

$$\begin{aligned}\dot{x}(t) &= Ax(t) + B_1u(t) + B_2w(t), \quad x(0) = x_0,\\ y(t) &= x(t),\end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the control input, $y \in \mathbb{R}^q$ represents outputs, and $w(t) \in \mathbb{R}^l$ is zero-mean white Gaussian process noise with covariance W. The feedback loop is closed over a bandwidth-limited communication channel which is assumed to be free of time-delays and packet-drops.

The objective is to control system (1) such that the control cost, communication bandwidth, and computational load are optimised. Following this, the controller is based on LQ design and the states are transmitted using ST sampling, due to which (1) becomes sampled-data system, given by

$$\begin{aligned}\dot{x}(t) &= Ax(t) + B_1u(t) + B_2w(t), \quad x(0) = x_0,\\ y(\delta_i) &= x(\delta_i),\\ u(t) &= f(x(\delta_i)), \quad \forall t \in [\delta_i, \delta_{i+1}),\end{aligned} \tag{2}$$

where $\delta_i \in \mathbb{R}_{\{0,+\}}$ represents the sampling time with $i \in \mathbb{Z}_{\{0,+\}}$ and $f(.) : \mathbb{R}^n \to \mathbb{R}^m$ is a causal mapping which optimises certain cost function.

3 ST LQ regulator [23]

A recently proposed ST LQ regulation technique [23] is reproduced briefly. The co-design problem is solved for (2) with the idea of maximising the sampling interval while guaranteeing performance based on discounted LQ cost

$$J = \sum_{t=0}^{\infty} \mathbb{E}[\gamma^t (x_t Q x_t + u_t^{\mathrm{T}} R u_t^{\mathrm{T}} + 2x_t^{\mathrm{T}} H u_t)|x_0], \tag{3}$$

where $0 < \gamma < 1$ represents the discount factor and Q, R, and H represent the standard weighting matrices. Particularly, the co-design problem is stated as

$$\begin{aligned}t_{l+1} &= t_l + M(x_{t_l}),\\ u_t &= \bar{u}_l \in \mathcal{U}_M(x_{t_l}), \quad t \in \mathbb{Z}_{\{0,+\}}\end{aligned} \tag{4}$$

where $t_l \in \mathbb{R}_{\{0,+\}}$ represents the sampling time at sample number $l \in \mathbb{Z}_{\{0,+\}}$. $M : \mathbb{R}^n \to \{1, \ldots, \bar{M}\}$ is the sampling interval with $\bar{M} \in \mathbb{Z}_+$ arbitrarily large for bounded latency, and $\mathcal{U} : \mathbb{R}^n \to \mathbb{R}^m$ is a set-valued map representing the set of possible control values.

To present M and $\mathcal{U}$ in (4), $\mathcal{U}_M(x)$ is defined, for $x \in \mathbb{R}^n$, as the set of control values that can be held constant for M steps while satisfying

$$\mathbb{E}\left[\left(\sum_{t-t_l}^{t_{l+1}-1} \gamma^{t-t_l}(x_j^{\mathrm{T}} Q x_j + \bar{u}_j^{\mathrm{T}} R \bar{u}_j + 2x_j^{\mathrm{T}} H \bar{u}_j)\right) + \gamma^{t_{l+1}-t_l} V_{\beta_1,\beta_2}(x_{t_{l+1}})|x_{t_l}\right] \le V_{\beta_1,\beta_2}(x_{t_l}), \tag{5}$$

at transmission time t_l, where

$$V_{\beta_1,\beta_2}(x) := \beta_1 x^{\mathrm{T}} P x + \beta_2 \frac{\alpha}{1-\alpha} \mathrm{tr}\left(PB_2B_2^{\mathrm{T}}\right), \tag{6}$$

with P as the solution of following discrete algebraic Riccati equation

$$P = Q + \gamma A^{\mathrm{T}} P A - \left(\gamma A^{\mathrm{T}} P B_1 + H\right)\left(R + \gamma B_1^{\mathrm{T}} P B_1\right)^{-1} \times \left(\gamma B_1^{\mathrm{T}} P A + H^{\mathrm{T}}\right). \tag{7}$$

This leads to

$$\mathcal{U}_M(x) := \left\{\bar{u} \in \mathbb{R}^n | \mathbb{E}\left[\left(\sum_{j=0}^{M-1} \gamma^j (\bar{x}_j^{\mathrm{T}} Q \bar{x}_j + \bar{u}_j^{\mathrm{T}} R \bar{u}_j + 2\bar{x}_j^{\mathrm{T}} H \bar{u}_j)\right) + \gamma^M V_{\beta_1,\beta_2}(\bar{x}_M)|x\right] \le V_{\beta_1,\beta_2}(x)\right\}, \tag{8}$$

where $\bar{x}_j, j \in \{1, 2, \ldots, M\}$, is the solution of the discretised version of $\dot{x} = Ax(t) + B_1u(t) + B_2w(t)$ with $\bar{x}_0 = x$ and $u_t = \bar{u}$, $t \in \mathbb{Z}$, i.e.

$$\bar{x}_j = \bar{A}_j x + \bar{B}_{1,j}\bar{u} + \bar{B}_{2,j}^M w_M, \tag{9}$$

where $\bar{A}_j := A^j$, $\bar{B}_{1,j} := \sum_{i=0}^{j-1} A^i B_1$, and $\bar{B}_{2,j}^M \in \mathbb{R}^{n \times Ml}$ is given as

$$\bar{B}_{2,j}^M := [A^{j-1}B_2 \ \cdots \ AB_2 \ B_2 \ 0 \ \cdots \ 0] \tag{10}$$

and $w_M := [w_0^{\mathrm{T}}, w_1^{\mathrm{T}}, \ldots, w_{M-1^{\mathrm{T}}}]^{\mathrm{T}}$.

From above discussion, it can be seen that $\mathcal{U}_M(x) \neq \emptyset$ if and only if

$$\min_{\bar{u}\in\mathbb{R}^n} \mathbb{E}\left[\left(\sum_{j=0}^{M-1} \gamma^j (\bar{x}_j^{\mathrm{T}} Q\bar{x}_j + \bar{u}_j^{\mathrm{T}} R\bar{u}_j + 2\bar{x}_j^{\mathrm{T}} H\bar{u}_j)\right)\right.$$
$$\left. + \gamma^M V_{\beta_1,\beta_2}(\bar{x}_M)|x\right] \le V_{\beta_1,\beta_2}(x). \quad (11)$$

Using (6) and (9), the above equation becomes

$$\min_{\bar{u}\in\mathbb{R}^n} \mathbb{E}\left[\bar{x}^{\mathrm{T}} F_M \bar{x} + \bar{x}^{\mathrm{T}} G_M \bar{u} + \frac{1}{2}\bar{u}^{\mathrm{T}} U_M \bar{u} + c_M\right] \le V_{\beta_1,\beta_2}(x), \quad (12)$$

where

$$F_M = \gamma^M \beta_1 \bar{A}_M^{\mathrm{T}} P \bar{A}_M + \sum_{j=0}^{M-1} \gamma^j \bar{A}_j^{\mathrm{T}} Q \bar{A}_j,$$
$$G_M = 2\left[\gamma^M \beta_1 \bar{A}_M^{\mathrm{T}} P \bar{B}_M + \sum_{j=1}^{M-1} \gamma^j (\bar{A}_j^{\mathrm{T}} Q \bar{B}_j + \bar{A}_j^{\mathrm{T}} H)\right],$$
$$U_M = 2\left[\gamma^M \beta_1 \bar{B}_M^{\mathrm{T}} P \bar{B}_M + \sum_{j=0}^{M-1} \gamma^j (\bar{B}_j^{\mathrm{T}} Q \bar{B}_j + \bar{B}_j^{\mathrm{T}} H + H^{\mathrm{T}} \bar{B}_j + R)\right],$$
$$c_M = d_M + \beta_2 \alpha^M \frac{\alpha}{1-\alpha} \mathrm{tr}\left(P B_2 B_2^{\mathrm{T}}\right),$$
$$d_M = \gamma^M \beta_1 \mathrm{tr}\left(P \bar{B}_{2,M}^M \left(\bar{B}_{2,M}^M\right)^{\mathrm{T}}\right) + \sum_{j=1}^{M-1} \gamma^j \mathrm{tr}\left(Q \bar{B}_{2,j}^M \left(\bar{B}_{2,j}^M\right)^{\mathrm{T}}\right). \quad (13)$$

The optimal control input

$$\bar{u}^* := \mathrm{argmin}_{\bar{u}\in\mathcal{U}_M(x)} x^{\mathrm{T}} F_M x + x^{\mathrm{T}} G_M \bar{u} + \frac{1}{2}\bar{u}^{\mathrm{T}} U_M \bar{u} + c_M$$

is obtained by solving

$$\frac{\partial}{\partial \bar{u}}\left(x^{\mathrm{T}} F_M x + x^{\mathrm{T}} G_M \bar{u} + \frac{1}{2}\bar{u}^{\mathrm{T}} U_M \bar{u} + c_M\right) = 0,$$

which leads to $x^{\mathrm{T}} G_M + \bar{u}^{\mathrm{T}} U_M = 0$, and thus

$$\bar{u}^* = -U_M^{-1} G_M^{\mathrm{T}} x =: K_M x, \quad (14)$$

and

$$M(x) = \max\left\{M \in \{1, 2, \ldots, \bar{M}\} | x^{\mathrm{T}} P_M^* x + \bar{c}_M \le \beta_1 x^{\mathrm{T}} P x\right\}, \quad (15)$$

where $P_M^* = F_M - \frac{1}{2} G_M U_M^{-1} G_M^{\mathrm{T}}$ and $\bar{c}_M = d_M - \beta_2 \sum_{j=1}^{M} \gamma^j \mathrm{tr}(P B_2 B_2^{\mathrm{T}})$. Hence the co-design problem, given by (4) is solved with the control input (14) for the sampling time computed as (15).

4 Proposed methodology

The scheme reported in [23] does not consider communication delays, decoupled design of control and triggering mechanism, and optimisation of computational resources. Therefore, a novel ST LQ control methodology is proposed and presented here. This approach is based on bounding the triggering interval between $\alpha \in \mathbb{R}_+$ and $\Gamma\alpha$, where the choice of α depends on the plant dynamics, and $1 \le \Gamma \in \mathbb{Z}$ ensures bounded latency, which is the same as $\bar{M}$ in [23]. These bounds enable separate synthesis of controller and ST sampler, breaking the controller module into two routines, one for the *asynchronous LQ regulator* and second for the ST sampler. It depends on the designer to implement the controller on single or two independent computing platforms (one for each routine), in which case the computation time will decrease at an increased implementation cost. Here it is assumed that both these routines are implemented on single machine.

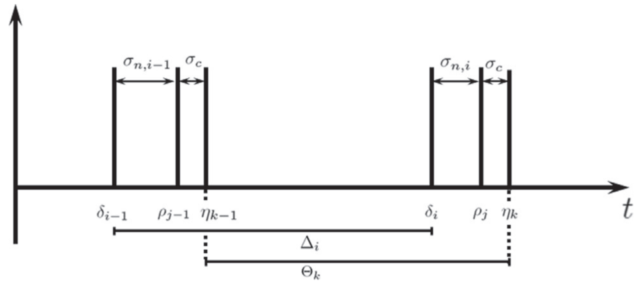

Fig. 1 *Timing diagram illustrating sampling (δ), reception (ρ), and control application (η) times with transmission and computation delays*

Remark 1: This approach employs the asynchronous LQ regulator which can incorporate time-delays. However, the case studies in this paper consider perfect communication channel to present a fair comparison against [23], which does not consider the delays.

4.1 Asynchronous LQG controller

Let the sampling time provided to the sensor node by the ST sampler be δ_i (see Fig. 1). The state transmitted at δ_i reaches the controller at $\rho_j \in \mathbb{R}_+$, where $j \in \mathbb{Z}_{\{0,+\}}$, after encountering a random-but-bounded time-delay induced by the network, $\sigma_{n,i} \le \overline{\sigma}_n \in \mathbb{R}_+ \; \forall i$. The controller takes $\sigma_c \in \mathbb{R}_+$ units of time to estimate the state and compute the controller output u which will be applied at $\eta_k \in \mathbb{R}_+$, where $k \in \mathbb{Z}_{\{0,+\}}$, and $\eta_k = \rho_j + \sigma_c$. The difference between consecutive events is denoted as $\Delta_i \triangleq \delta_i - \delta_{i-1}$ and that between the corresponding consecutive controller updates is represented by $\Theta_k \triangleq \eta_k - \eta_{k-1}$. Note that $i = j = k$ translates into the ith event, and the corresponding jth reception and kth control update. Following these notations, $\underline{\Delta}, \underline{\Theta} \triangleq \alpha$ and $\overline{\Delta}, \overline{\Theta} \triangleq \Gamma\alpha$.

Assumption 1: It is assumed that:

1. $\delta_0, \rho_0, \eta_0 = 0$,
2. Computational delay σ_c is constant,
3. Received states are time-stamped, i.e. δ_i is received along with the ith transmission,
4. Delay $\sigma_{n,i} + \sigma_c$ is less than Δ_i, which implies that $\delta_i < \eta_k$, $\forall i = k$.

Due to this sampling scheme, (1) can be viewed as a *dual-rate* ASDS because the sampling interval at the sensor node is different from that of the control update, i.e. $\Delta_i \neq \Theta_k \; \forall i = k$. Hence

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_1 u(t) + B_2 w(t), \quad x(0) = x_0, \\ y(\delta_i) &= x(\delta_i), \\ u(t) &= u_{\eta_k} = f(\hat{x}(\delta_i)), \quad \forall t \in [\eta_k, \eta_{k+1}), \end{aligned} \quad (16)$$

where $f(.) : \mathbb{R}^n \to \mathbb{R}^m$ is a causal mapping which optimises certain cost function, and $\hat{x}(\delta_i) \in \mathbb{R}^n$ represents the estimated state.

To the best of the authors' knowledge, the solution of optimal control problem of ASDS [24] is applied for the first time on aperiodically-triggered systems. The motivation behind adopting ASDS approach is to reduce the complexity of design and analysis, particularly when the communication channel is imperfect. In [24], it was assumed that the sampling, and consequently the hold rates are fixed, however, in the case under consideration, the sampling time is aperiodic which renders the time of control application aperiodic. The sampling and hold times are translated into event and control update instants, respectively.

Assumption 2: It is assumed $\forall\, i, k$ that:

1. For DT systems, the pairs (C, A_{Δ_i}) are detectable,
2. Similarly, the pairs $(A_{\Theta_k}, B_{\Theta_k})$ are stablisable,
3. System (16) is stablisable and detectable,

where $A_a = e^{A(a)}$ and $B_a = \int_0^a A_s B_1 ds$ are the discretised system matrices for any time-interval a.

Remark 2: Assumption 2-(1,2) are necessary to ensure that, at every sampling interval the system is both detectable and stablisable when it is discretised for asynchronous linear quadratic Gaussian (LQG) implementation. Furthermore, Assumption 2-(3) ensures stablisability and detectability for the sampled-data system (16).

Let $\hat{x}_\tau$, x_τ, and u_τ represent the estimated state, sampled-state, and control input at time τ, respectively, and $Q_\tau \in \mathbb{R}^{n\times n} > 0$, $R_\tau \in \mathbb{R}^{m\times m} > 0$ and $H_\tau \in \mathbb{R}^{n\times m}$ denote, respectively, the state, input, and cross weighting matrices at τ, then the controller which minimises the discrete cost function

$$J_\Theta = \sum_{k=0}^{K-1} \frac{1}{K\eta_k} \mathbb{E}\left[x_{\eta_k}^{\mathrm{T}} Q_{\eta_k} x_{\eta_k} + u_{\eta_k}^{\mathrm{T}} R_{\eta_k} u_{\eta_k} + 2 x_{\eta_k}^{\mathrm{T}} H_{\eta_k} u_{\eta_k}\right] \tag{17}$$

is given as

$$\begin{aligned} u_{\eta_k} &= -\left(R_{\eta_k} + B_{1,\Theta_k}^{\mathrm{T}} P_k B_{1,\Theta_k}\right)^{-1} \left(B_{1,\Theta_k}^{\mathrm{T}} P_k A_{\Theta_k} + H_{\eta_k}^{\mathrm{T}}\right) \hat{x}_{\eta_k} \\ &= -\mathcal{K}_{\eta_k} \hat{x}_{\eta_k}, \end{aligned} \tag{18}$$

where the state estimate $\hat{x}_{\eta_k}$ is given as

$$\begin{aligned} \hat{x}_{\eta_k} &= A_{\eta_k - \delta_i} \hat{x}_{\delta_i} + B_{1,\eta_k-\delta_i} u_{\eta_{k-1}}, \quad \forall k = i, \\ \hat{x}_{\eta_0} &= x_0. \end{aligned} \tag{19}$$

The weighting matrices are given as

$$\begin{aligned} Q_{\eta_k} &= \int_{\eta_{k-1}}^{\eta_k} \left[A_{s-\eta_{k-1}}^{\mathrm{T}} Q A_{s-\eta_{k-1}}\right] \mathrm{d}s, \\ R_{\eta_k} &= \int_{\eta_{k-1}}^{\eta_k} \left[R + B_{1,s-\eta_{k-1}}^{\mathrm{T}} Q B_{1,s-\eta_{k-1}}\right] \mathrm{d}s, \\ H_{\eta_k} &= \int_{\eta_{k-1}}^{\eta_k} \left[A_{s-\eta_{k-1}}^{\mathrm{T}} Q B_{1,s-\eta_{k-1}}\right] \mathrm{d}s, \end{aligned} \tag{20}$$

which are discrete equivalents of Q and R in CT. $\{P_k\}$ is given by the unique positive semi-definite solution of DT Riccati equation

$$\begin{aligned} P_k &= A_{\Theta_k}^{\mathrm{T}} P_{k+1} A_{\Theta_k} - \left(A_{\Theta_k}^{\mathrm{T}} P_{k+1} B_{1,\Theta_k} + H_{\eta_k}\right) \\ &\quad \times \left(R_{\eta_k} + B_{1,\Theta_k}^{\mathrm{T}} P_{k+1} B_{1,\Theta_k}\right)^{-1} \left(B_{1,\Theta_k}^{\mathrm{T}} P_{k+1} A_{\Theta_k} + H_{\eta_k}^{\mathrm{T}}\right) + Q_{\eta_k}, \\ P_K &= 0. \end{aligned} \tag{21}$$

According to (19), the estimated state for time instant η_k depends upon the state estimate for δ_i, which is given as

$$\hat{x}_{\delta_i} = \hat{x}_{\delta_i^-} + S_i [I + S_i]^{-1} \left(y_{\delta_i} - \hat{x}_{\delta_i^-}\right), \tag{22}$$

where

$$\begin{aligned} \hat{x}_{\delta_i^-} &= A_{\delta_i - \eta_{k-1}} \hat{x}_{\eta_{k-1}} + B_{1,\delta_i-\eta_{k-1}} u_{\eta_{k-1}}, \\ \hat{x}_{\delta_0^-} &= x_0, \end{aligned} \tag{23}$$

and $\{S_i\}$ is the solution of following Riccati equation

$$\begin{aligned} S_i &= A_{\Delta_i} S_{i-1} A_{\Delta_i}^{\mathrm{T}} - A_{\Delta_i} S_{i-1} [I + S_{i-1}]^{-1} S_{i-1} A_{\Delta_i}^{\mathrm{T}} + W_{\delta_i}, \\ S_0 &= 0, \end{aligned} \tag{24}$$

with $W_{\delta_i} = \int_{\delta_{i-1}}^{\delta_i} A_{\delta_i - t} B_2 B_2^{\mathrm{T}} A_{\delta_i - t}^{\mathrm{T}}\, \mathrm{d}t$.

However, the solution of (21) requires the knowledge of P_{k+1} which cannot be determined offline due to aperiodic triggering. Also the time-varying controller (18) is computationally intensive which translates into larger σ_c. These problems require a time-invariant controller, and the best choice for a constant control gain is to design it using the *worst-case* sampling time defined from control perspective, i.e. $\overline{\Theta}\ \forall k$, and optimisation done over infinite horizon which render (17) as

$$J_\Theta = \lim_{K\to\infty} \sum_{k=0}^{K-1} \frac{1}{K\eta_k} \mathbb{E}\left[x_{\eta_k}^{\mathrm{T}} Q_{\overline{\Theta}} x_{\eta_k} + u_{\eta_k}^{\mathrm{T}} R_{\overline{\Theta}} u_{\eta_k} + x_{\eta_k}^{\mathrm{T}} H_{\overline{\Theta}} u_{\eta_k}\right]. \tag{25}$$

This assumption allows the designer to compute a constant control gain

$$\mathcal{K}_{\overline{\Theta}} = \left(R_{\overline{\Theta}} + B_{1,\overline{\Theta}}^{\mathrm{T}} P B_{1,\overline{\Theta}}\right)^{-1} \left(B_{1,\overline{\Theta}}^{\mathrm{T}} P A_{\overline{\Theta}} + H_{\overline{\Theta}}^{\mathrm{T}}\right), \tag{26}$$

offline, such that

$$u(t) = -\mathcal{K}_{\overline{\Theta}} \hat{x}(\eta_k), \quad \forall t \in [\eta_k, \eta_{k+1}), \tag{27}$$

and P is the solution of following algebraic Riccati equation (ARE)

$$\begin{aligned} P &= A_{\overline{\Theta}}^{\mathrm{T}} P A_{\overline{\Theta}} - \left(A_{\overline{\Theta}}^{\mathrm{T}} P B_{1,\overline{\Theta}}\right) \left(R_{\overline{\Theta}} + B_{1,\overline{\Theta}}^{\mathrm{T}} P B_{1,\overline{\Theta}}\right)^{-1} \\ &\quad \times \left(B_{1,\overline{\Theta}}^{\mathrm{T}} P A_{\overline{\Theta}} + H_{\overline{\Theta}}^{\mathrm{T}}\right) + Q_{\overline{\Theta}}. \end{aligned} \tag{28}$$

Remark 3: This choice of controller will ensure that the actual cost of the aperiodic system is lower than (or at most equal to) the cost paid for the worst-case system.

To maintain accuracy the estimation is done using (19), (22)–(24). To further reduce the computational time, the integral in (24) is computed offline for interval α, i.e. W_α. Then, during the real-time estimation, it is multiplied with the number of α-spaced intervals in Δ_i to get W_{δ_i}.

4.2 ST sampler

The novel ST sampling method presented here is based on the fact that an exponential relationship exists between the difference of costs of DT and CT systems, and the sampling frequency f [25]. Let the difference be denoted as $\Delta J^*(f) \triangleq J_{\mathrm{DT}}^*(f) - J^*$, where $J_{\mathrm{DT}}^*(f)$ represents the optimal cost of the discrete system sampled periodically at f, and J^* gives the optimal cost of the CT system. The relationship is thus given by

$$\Delta J^*(f) = a e^{-bf}, \tag{29}$$

where $a, b \in \mathbb{R}_+$ are system dependant constants determined by simulating the plant at several sampling frequencies in the desired range. Recently, this relationship was used in [26] for offline optimisation of control costs and *periodic* transmission frequencies of multiple systems sharing the same wireless network.

The objective of this work is to design a ST sampler which can predict the *cost-dependant sampling frequency* online. The idea is to keep the control cost of the ST system as close to the reference system as possible, while ensuring communication-bandwidth economy, especially when the ST system is regulated. The reference system is sampled-data implementation of the plant sampled periodically at every α units of time, with a perfect communication channel. In particular, the reference system is given as

$$\begin{aligned}\dot{x}(t) &= Ax(t) + B_1 u(t) + B_2 w(t), \quad x(0) = x_0,\\ y(t) &= x(t),\\ u(t) &= -\mathcal{K}_\alpha x(q\alpha) \ \ \forall t \in [q\alpha, (q+1)\alpha),\end{aligned} \tag{30}$$

where $q \in \mathbb{Z}_{\{0,+\}}$, and it is assumed that W is known.

Remark 4: Note that instead of taking the CT feedback implementation as a reference, α-periodic system is considered. This is due to the over stringent demand on the control cost if the ST system's cost is compared against CT implementation, which will translate into high sampling frequencies.

Remark 5: When the cost difference is taken against the reference system, it follows the same relation as (29).

To design the ST sampler, the sampling frequency range is defined by the bounds on sampling time, i.e. $\alpha \le \Delta_i \le \Gamma\alpha \ \forall i$. The minimum sampling frequency $f_m \triangleq (1/\Gamma\alpha)$ and maximum is given as $f_M \triangleq (1/\alpha)$. For several sampling frequencies in the range $[f_m, f_M]$, the respective controllers are computed and control cost difference at each of these frequencies is obtained as

$$\Delta J^*(f) = x_0^{\mathrm{T}}(P_f - P_{f_M})x_0, \tag{31}$$

where P_f is defined for any sampling frequency in the selected range, and x_0 is the initial state of the plant assumed to be known. During the operation of ST system, the *cost-to-go* from $\delta_i \ \forall \ i$ is obtained as $J_{\mathrm{ST}}|_{\delta_i} \triangleq x_{\delta_i}^{\mathrm{T}} P_{f_m} x_{\delta_i}$ due to the static controller, and the cost difference is computed as

$$\Delta J_{\mathrm{ST}}|_{\delta_i} \triangleq J_{\mathrm{ST}}|_{\delta_i} - J_{f_M}^*|_{\delta_i} = x_{\delta_i}^{\mathrm{T}} P_{f_m} x_{\delta_i} - x_{\delta_i}^{\mathrm{T}}|_\alpha P_{f_M} x_{\delta_i}|_\alpha, \tag{32}$$

where x_{δ_i} and $x_{\delta_i}|_\alpha$ represent the states of the ST and reference systems at δ_i, respectively. The cost evolution of the reference system is simulated offline and stored in the ST setup.

Note that the variation in cost difference (31) is due to the change in the sampling frequency which in turn changes P_f. However, in (32) the variation will only result from the change in x_{δ_i}. This problem requires to translate $\Delta J_{\mathrm{ST}}|_{\delta_i}$ into (31). Therefore, the cost difference (31) is normalised with respect to $x_0^{\mathrm{T}}(P_{f_m} - P_{f_M})x_0$. After normalisation, the $\Delta J^*(f)$ against f graph is scaled from 0 to 1 for $[f_m, f_M]$ and an exponential relationship is obtained using curve-fitting which gives the values of the parameters a and b, as shown in Fig. 2, where $\mathcal{C}^f$ represents the fitted curve. During the course of operation, the cost difference of ST system (32) is also normalised with $x_0^{\mathrm{T}}(P_{f_m} - P_{f_M})x_0$ at $\delta_i \ \forall i$.

The desired behaviour of the ST sampler should be to choose a high sampling frequency when the cost of the ST system is far from that of the reference system and vice versa. Solving a relationship similar to (29) for frequency cannot fulfil this requirement, necessitating a mechanism whereby the sampler *reacts* to the change in ΔJ_{ST} at every δ_i to imitate the desired behaviour. To achieve this, following requirements are listed:

- At maximum cost difference of the ST system, the sampling frequency should be maximum,
- When the difference is close to zero, the sampling frequency should be close to minimum,
- For a sharp increase (decrease) in the cost difference, the sampling frequency should be *immediately* increased (decreased),
- When choosing the minimum sampling frequency (corresponding to $\overline{\Delta}$), the sampler should take care of the network induced delay.

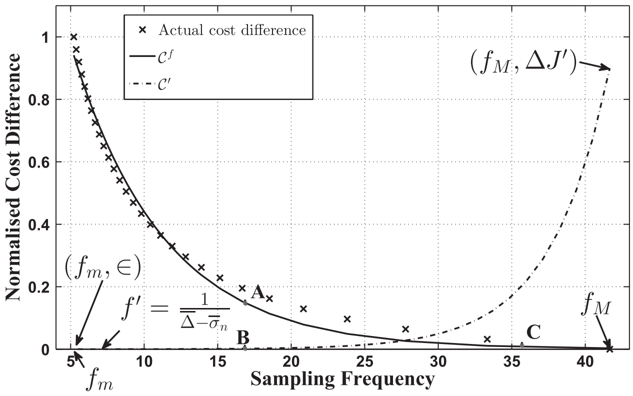

Fig. 2 *ST sampler synthesis*

Following these requirements, another curve $\mathcal{C}'$ is introduced on ΔJ against f graph, as shown in Fig. 2, which is an exponentially rising function of the form

$$\mathcal{C}' \triangleq h e^{pf}, \tag{33}$$

defined over $f \in [f_m, f_M]$ with the parameters $h, p \in \mathbb{R}_+$ determined with the knowledge of two points resulting from the requirements stated above. The first point has the coordinates (f_m, ϵ) and the second is located at $(f_M, \Delta J')$, where $\epsilon = ae^{-bf_M}$ and $\Delta J'$ is the cost difference on $\mathcal{C}'$ at frequency $f' = (1/\overline{\Delta} - \overline{\sigma}_n)$ to tackle the network induced delay. With these two points, (33) is solved which yields h and p.

The ST sampler will compute next triggering time following these steps:

i. Given the sampled state of the plant x_{δ_i}, the normalised cost difference is computed as

$$\Delta J_{\mathrm{ST}}|_{\delta_i, N} = \frac{\Delta J_{\mathrm{ST}}|_{\delta_i}}{x_0^{\mathrm{T}}(P_{f_m} - P_{f_M})x_0}. \tag{34}$$

ii. Given $\Delta J_{\mathrm{ST}}|_{\delta_i, N}$, (29) is solved for frequency (point A in Fig. 2), i.e.

$$f_1 = -\frac{1}{b}\ln\left(\frac{\Delta J_{\mathrm{ST}}|_{\delta_i, N}}{a}\right). \tag{35}$$

iii. With f_1, (33) is solved for $\Delta J_N'$ (point B), i.e.

$$\Delta J_N' = h e^{pf_1}. \tag{36}$$

iv. With $\Delta J_N'$, (29) is again solved to compute the sampling frequency (point C) as

$$f_{\mathrm{s}} = -\frac{1}{b}\ln\left(\frac{\Delta J_N'}{a}\right). \tag{37}$$

v. The next triggering time δ_{i+1} is computed as

$$\delta_{i+1} = \delta_i + \frac{1}{f_{\mathrm{s}}}. \tag{38}$$

These steps are repeated every time the state is received.

Remark 6: Stability due to the proposed ST sampling scheme is guaranteed owing to Assumption 2 and the choice of sampling frequency in a predefined range $[f_m, f_M]$ which is based on system's dynamics.

5 Comparison

In this section, the proposed methodology is compared with [23] from three perspectives: (i) controller design, (ii) ST sampling, and (iii) computational requirements, and simulation results are presented for three case studies.

5.1 Controller design

The controller presented in [23] does not incorporate time-delays, which makes the scheme unsuitable for implementation over real-time networks. In contrast, the proposed approach can tackle computational and communication delays by estimating the state for the time when control input will be actually applied, given the state at the transmission time. Furthermore, the choice of static control simplifies the design.

Regarding the control cost, both approaches use similar idea of keeping the cost close to periodic system. Particularly, in [23] the cost of ST system is compared against *scaled* (by β_1) cost of the reference system (15), and in proposed scheme, the sampler increases frequency if the cost difference between ST and reference systems increases.

5.2 ST sampling

The novel sampler reacts to the change in cost of the ST system such that, the further the cost of ST system goes from that of the reference system, the higher the sampling frequency becomes, and vice versa. Consequently, the sampling intervals in the transient period are much less than those in the regulated period. On the other hand, the sampler in [23] based on condition (15), results in higher average sampling interval in the transient period than that during regulation. In general, this leads to higher communication cost than the proposed scheme.

5.3 Computational requirements

For the case without time-delays, computational time for the proposed scheme is less than that for [23] due to the static gain. However, when delays are considered, the computational time increases as the asynchronous controller estimates the state for η_k.

When comparing the sampling methodology, computational time of proposed sampler is less than that in [23]. However, the memory requirements are higher because of the storage of cost evolution of the reference system.

5.4 Case studies

Simulation study of above defined approaches is presented for mass-spring, water-level control and inverted-pendulum over cart systems, which offer both slow and fast dynamics. For a fair comparison, a perfect communication channel is assumed, and same values of weighting matrices are taken for both approaches. The clock speed and precision of the computational platform are 3300 MHz and 30 ns, respectively. The results are analysed on the basis of control cost, communication bandwidth usage in terms of event-rate (number of events per second), and computational time.

5.4.1 Mass-spring system: The system, described in [23], consists of two masses ($m_1 = m_2 = 1$) connected by spring and damper. It is represented by (1) with $x = [y_1\ y_2\ \dot{y}_1\ \dot{y}_2]^{\mathrm{T}}$ and

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k_s & k_s & -d & d \\ k_s & -k_s & d & -d \end{bmatrix};\quad B_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix};\quad B_2 = \begin{bmatrix} 0 \\ 0 \\ 0.02 \\ 0 \end{bmatrix}, \tag{39}$$

where $k_s = 5$ and $d = 1$ are the spring and damper coefficients. The initial state $x_0 = [0.49\ {-0.4}\ 0.74\ {-0.25}]^{\mathrm{T}}$ and $W = 1$. The value of α is chosen on the basis of dominant pole as $(1/5\lambda_{\mathrm{dom}})$, where λ_{dom} represents the dominant pole; for this system α = 63.2 ms and the upper-bound $\Gamma = 5$. For the proposed approach, the results of ST sampler synthesis are given in Fig. 3.

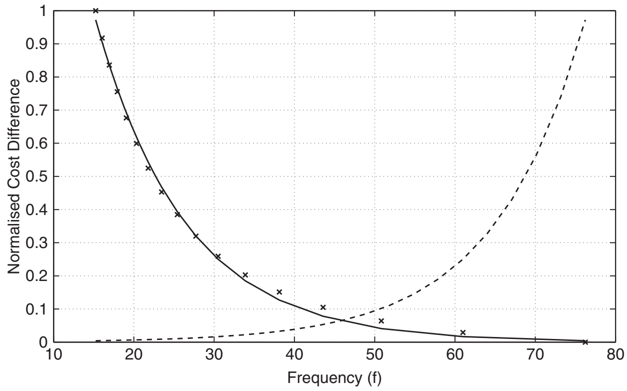

Fig. 3 *ST sampler synthesis for mass-spring system*

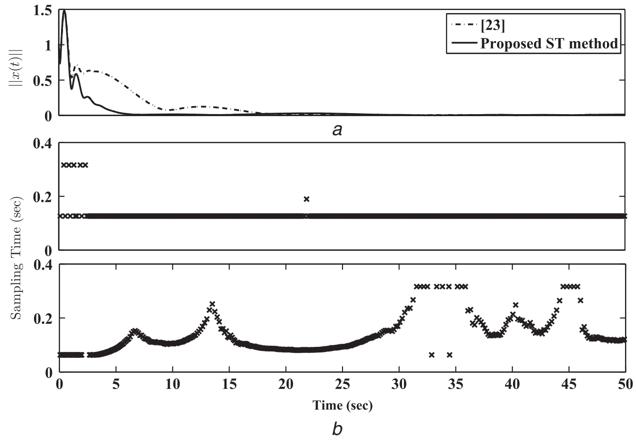

Fig. 4 *State response and sampling times for both methodologies*

a State norm
b Sampling times for mass-spring system. Top: Gommans *et al.* [23]; bottom: novel ST scheme

For [23], $\beta_1 = 1.1$, $\beta_2 = 1.5$, $\gamma = 0.99$, $Q = I^{(4)}$, and $R = 1$. The weighting matrices are discretised for sampled-data implementation using (20) with period α for the reference system and [23], while $\Gamma\alpha$ is used for the proposed method.

The control costs for [23] and proposed scheme are 5.48 and 5.29, respectively. Fig. 4*b* presents the sampling times for both methodologies, and the performance of novel approach can be clearly observed. During the period when states are regulated, the sampling time increases which results in lesser number of events, translating into communication bandwidth economy. Specifically, the event rate for [23] is 15.8 which is greater than 8.58 for the proposed scheme.

The computation time of the new approach is 0.016 ms, while that for [23] is 0.02 ms. Conclusively, the proposed methodology results in better performance than *I* with 3.5, 44, and 20% reduction in control cost, event-rate, and computational time, respectively.

5.4.2 Four-tank system: The process, given in [27], contains four tanks and two pumps, and the objective is to control the water level in tanks 1 and 2. The pump flows are divided by valves and their position determines the location of system's zeroes, making the system minimum or non-minimum phase. In this analysis, only minimum phase case is considered, and that all the states are measurable. Table 1 gives the parameters and description.

The states of linearised system (1) are $x_i := h_i - h_{\mathrm{ss}}$ for $i \in \{1, 2, 3, 4\}$, inputs $u := v_j - v_{j,\mathrm{ss}}$ where v_j represents the voltage input of the jth pump with $v_{j,\mathrm{ss}}$ as its steady-state value and $j \in$

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Table 1 Parameters of the four-tank system

Parameter	Description	Value
h_{ss}	steady-state water level	15 cm
a	cross-section area of the tanks	15.52 cm
o	cross-section area of the outlet	0.178 cm
g	acceleration due to gravity	981 cm/s^2
k_p	pump flow constant	3.3 cm^3/sV
θ	valve flow ratio	0.75

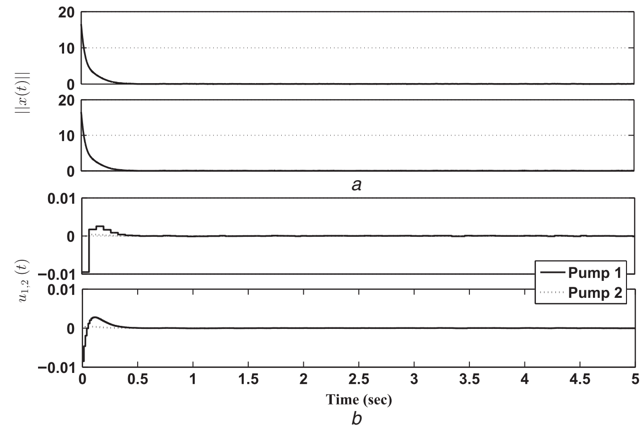

Fig. 5 *Norm of sate trajectories*

a State norm. Top: Gommans *et al.* [23]; bottom: novel ST scheme
b Inputs. Top: Gommans *et al.* [23]; bottom: novel ST scheme

$\{1, 2\}$, and

$$A = \begin{bmatrix} a_1 & 0 & a_2 & 0 \\ 0 & a_1 & 0 & a_2 \\ 0 & 0 & a_1 & 0 \\ 0 & 0 & 0 & a_1 \end{bmatrix}; \quad B_1 \begin{bmatrix} b_1 & 0 \\ 0 & b_1 \\ 0 & b_2 \\ b_2 & 0 \end{bmatrix}; \quad B_2 = \begin{bmatrix} 2 \\ 2 \\ 0.4 \\ 0.6 \end{bmatrix}, \tag{40}$$

where $a_1 = -(1/T)$ and $a_2 = (1/T)$ with $T = (o\sqrt{2g}/a)(1/2\sqrt{h_{ss}})$, and $b_1 = (\theta k_p/a)$ and $b_2 = (1 - \theta k_p/a)$. The initial state $x_0 = [10\ 5\ -7\ -10]^T$ and $W = 1$. The sampling period α is chosen as 13.1 ms and $\Gamma = 5$, which give the ST sampler similar to Fig. 3 with $f_m = 15.27$ Hz and $f_M = 76.34$ Hz.

The values of $\beta_1 = 1.5$, $\beta_2 = 1.1$, $Q = I^{(4)}$, and $R = 25I^{(2)}$. The norm of sate trajectories is shown in Fig. 5*a*, and both methodologies result in similar performance.

The control cost for [23] is 167.48 and that for the new approach is 111.15; the reason for this difference is slightly greater amount of control effort exerted by the controller of [23] in the transient period as demonstrated by Fig. 5*b*. Fig. 6 demonstrates the behaviour of samplers, and rates for [23] and the proposed scheme are 65 and 52.8.

Conclusively, the proposed method gives 33.6 and 18.76% decrease in control and communication costs as compared with [23]. The computational times are similar to that of the mass-spring system.

5.4.3 Pendulum-cart system: The system is described in [14] with states $x = [y\ \dot{y}\ \phi\ \dot{\phi}]^T$, where y and ϕ denote cart's position and bob's angle, respectively. Bobs mass $m_b = 1$, the carts mass $M_c = 10$, length of the pendulum $l = 3$, gravitational acceleration $g = 10$, and $W = 1$; system represented by (1) is given as

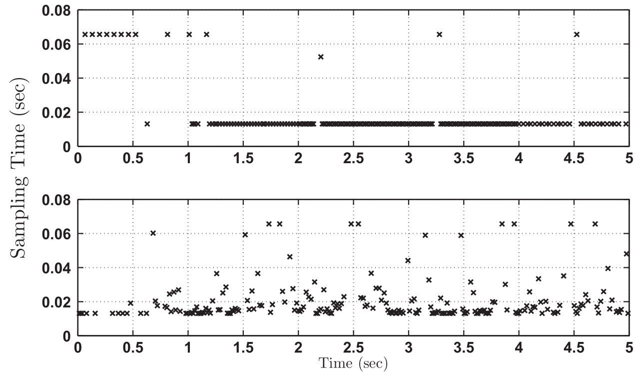

Fig. 6 *Sampling times for the four-tank system. Top: [23]; bottom: novel ST scheme*

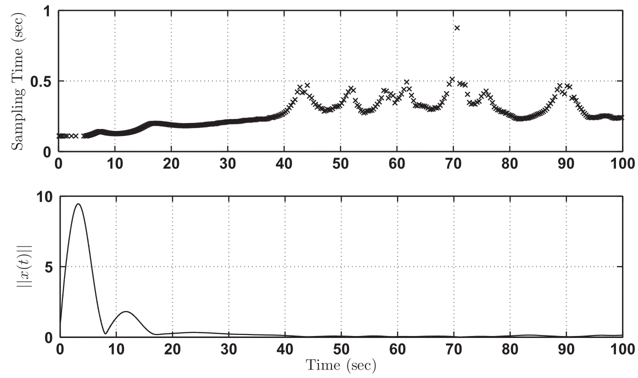

Fig. 7 *Top: sampling times; bottom: state norm*

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -\dfrac{m_b g}{M_c} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \dfrac{g}{l} & 0 \end{bmatrix}; \quad B_1 = \begin{bmatrix} 0 \\ \dfrac{1}{M_c} \\ 0 \\ -\dfrac{1}{M_c l} \end{bmatrix}; \quad B_2 = \begin{bmatrix} 0.02 \\ 0.02 \\ 0.02 \\ 0.02 \end{bmatrix}. \tag{41}$$

The values of $x_0 = [0.9\ 0\ 0.4\ 0]^T$, $\alpha = 109.5$ ms, $\Gamma = 8$, $\beta_1 = \beta_2 = 1.1$, $Q = 100I^{(4)}$, $R = 1$, and $\gamma = 0.99$. The results of ST sampler synthesis are similar to Fig. 3 with $f_m = 1.14$ Hz and $f_M = 9.13$ Hz.

For [23], the controller was unable to stabilise the system even for different values of $\beta_{1,2}$ and weighting matrices. In contrast, the proposed scheme gave a stable system with events and state norm shown in Fig. 7. The sensitivity of the proposed sampler can be observed as the sampling interval decreases even for a slight increase in the ST system's cost against the reference system. The event-rate, control cost, and computation time are 4.26, 9.12×10^4, and 0.012 ms, respectively.

6 Conclusion

A novel ST LQR scheme was presented which addresses the shortcomings in the existing literature, and a comparative study against [23] was presented with simulation results for three case studies. It can be concluded that the proposed approach is suitable for the case where control, communication, and computational resources are critical, and if the communication channel introduces delays then this scheme is the only choice. The simulation results showed superior control performance as compared with [23], which failed to stabilise the system in one of the cases. With regards to the sampling methodology, the proposed sampler results in significant communication-bandwidth economy than [23]. This is due to the fact that the sampler tries to achieve the same performance as the reference periodic system at much lower sampling frequency, particularly when the system is regulated.

In terms of the computational requirements, the proposed methodology takes less time as compared with [23]. However, the memory requirements are higher due to the storage of reference system's cost evolution. For limited memory applications and at the expense of increased computational time, a copy of the reference system may run in real-time to compare its cost against that of the ST system.

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